PROCEEDINGS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

VOL. 70 DECEMBER, 1944 NO. 10

TECHNICAL PAPERS

AND

DISCUSSIONS

Published monthly, except July and August, at Prince and Lemon Streets, Lancaster, Pa., by the American Society of Civil Engineers. Editorial and General Offices at 33 West Thirty-ninth Street, New York, N. Y. Reprints from this publication may be made on condition that the full title of Paper, name of Author, page reference, and date of publication by the Society, are given.

Entered as Second-Class Matter, September 23, 1937, at the Post Office at Lancaster, Pa., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 5, 1918.

Subscription (if entered before January 1) $8.00 per annum. Price $1.00 per copy

Copyright, 1944, by the AMERICAN SOCIETY OF CIVIL ENGINEERS

Printed in the United States of America

	Published	Discussion closes
Symposium: Rainbow Arch Bridge Over Niagara Gorge	Oct., 1943	
Discussion in Dec., 1943, Jan., Feb., Mar., Apr., May, June, 1944		Closed*
Edwards, F. W., and Soucek, Edward. Surges in Panama Canal Reproduced in Model	Jan., 1944	
Discussion in Apr., May, June, Dec., 1944		Closed
Huntting, Marshall T. Geology in Highway Engineering	Dec., 1943	
Discussion in Feb., Mar., Apr., May, June, Sept., Dec., 1944		Closed
Eisenlohr, William S., Jr. Coefficients for Velocity Distribution in Open-Channel Flow	Jan., 1944	
Discussion in Apr., Sept., 1944		Closed*
Symposium: Military Airfields	Jan., 1944	
Discussion in Feb., Mar., Apr., May, June, Sept., Nov., Dec., 1944		Closed*
Clark, C. O. Storage and the Unit Hydrograph	Nov., 1943	
Discussion in Feb., Mar., May, June, Sept., Dec., 1944		Closed*
Steinman, D. B. Rigidity and Aerodynamic Stability of Suspension Bridges	Nov., 1943	
Discussion in Feb., Mar., May, June, Sept., Dec., 1944		Closed*
Symposium: Unusual Cutoff Problems—Dams of the Tennessee Valley Authority	Nov., 1943	
Discussion in Feb., Apr., May, 1944		Closed*
Wilson, David M. Analysis of Rigid Frames by Superposition	Feb., 1944	
Discussion in May, June, Sept., Nov., 1944		Jan. 1, 1945
Gray, Hamilton. Simultaneous Consolidation of Contiguous Layers of Unlike Compressible Soils	Feb., 1944	
Discussion in June, Sept., 1944		Jan. 1, 1945
Komora, Andrew M. Erection of Hydraulic Turbines and Generators	Mar., 1944	
Discussion in June, Sept., 1944		Jan. 1, 1945
Kinnison, H. B., and Colby, B. R. Flood Formulas Based on Drainage Basin Characteristics	Mar., 1944	
Discussion in June, Sept., Oct., Nov., 1944		Jan. 1, 1945
Benscoter, Stanley U., and Logan, Samuel T. Shear and Bond Stresses in Reinforced Concrete	Mar., 1944	
Discussion in May, June, Sept., 1944		Jan. 1, 1945
Amster, Maurice N. Hydraulic Model Investigation of Miter Gate Operation	Mar., 1944	
Discussion in June, Sept., 1944		Jan. 1, 1945
Report: Bibliography on Timber Structures: Progress Report of the Committee of the Structural Division on Timber Structures	Mar., 1944	
Nubar, Yves. Analysis of the General Two-Dimensional Framework	Apr., 1944	
Discussion in June, Sept., 1944		Jan. 1, 1945
Report: Advance in Sewage Treatment and Present Status of the Art: Second Progress Report of the Committee of the Sanitary Engineering Division on Sewerage and Sewage Treatment	Apr., 1944	
Streeter, Victor L. Economical Canal Cross Sections	May, 1944	
Discussion in Sept., Oct., Nov., 1944		Feb. 1, 1945
Krynine, D. P. Pressure on the Lining of Circular Tunnels in Plastic Soils	May, 1944	
Discussion in Sept., Dec., 1944		Feb. 1, 1945
Foster, H. Alden. Construction of the Flow Net for Hydraulic Design	May, 1944	Feb. 1, 1945
Report: Design and Construction of Airport Runways and Flight Strips: Progress Report of a Committee of the Highway Division	May, 1944	Feb. 1, 1945
Vanoni, Vito A. Transportation of Suspended Sediment by Water	June, 1944	
Discussion in Dec., 1944		Feb. 1, 1945
Wilbur, John B. Characteristic Redundants Used for Analyzing Statically Indeterminate Structures	June, 1944	
Discussion in Sept., Dec., 1944		Feb. 1, 1945
Wilkinson, J. H. Translatory Waves in Natural Channels	June, 1944	Feb. 1, 1945
Moorman, Robert B. B. Wedge-Shaped Structural Members Under Direct Stress and Bending	June, 1944	
Discussion in Oct., 1944		Feb. 1, 1945
Terzaghi, Karl. Stability and Stiffness of Cellular Cofferdams	Sept., 1944	
Discussion in Dec., 1944		Feb. 1, 1945
Stewart, Ralph W. Amplified Slope Deflection	Sept., 1944	
Discussion in Nov., 1944		Feb. 1, 1945
Ahlf, August L. Design Constants for Beams with Nonsymmetrical Straight Haunches	Oct., 1944	Mar. 1, 1945
Soucek, Edward, and Zelnick, E. W. Lock Manifold Experiments	Oct., 1944	Mar. 1, 1945
Symposium: Concentration of Sewage Sludge	Oct., 1944	Mar. 1, 1945
Hickox, G. H. Evaporation from a Free Water Surface	Oct., 1944	Mar. 1, 1945
Bull, Anders. Stresses in the Linings of Shield-Driven Tunnels	Nov., 1944	Apr. 1, 1945
Johnson, Lee H., Jr. Analysis of Statically Indeterminate Structures Using Reduced Equations	Nov., 1944	Apr. 1, 1945

NOTE.—The closing dates herein published are final except when names of prospective discussers are registered for special extension of time.

* Publication of closing discussion pending.

CONTENTS FOR DECEMBER, 1944

PAPERS

PAGE

Flow in a Channel of Definite Roughness.
By Ralph W. Powell 1521

Conversion of Kinetic to Potential Energy in Flow Expansions.
By A. A. Kalinske 1545

Column Formulas.
By William R. Osgood 1565

DISCUSSIONS

Rigidity and Aerodynamic Stability of Suspension Bridges.
By Louis Balog 1573

Storage and the Unit Hydrograph.
By Don Johnstone 1585

Geology in Highway Engineering.
By Hyde Forbes, A. A. Eremin, and Marshall T. Huntting 1589

Surges in Panama Canal Reproduced in Model.
By F. W. Edwards and Edward Soucek 1598

Military Airfields: A Symposium.
By L. J. Mensch 1603

Characteristic Redundants Used for Analyzing Statically Indeterminate Structures.
By Ralph W. Stewart, Myle J. Holley, Jr., M. W. Huggins, and Leon Beskin 1609

Transportation of Suspended Sediment by Water.
By Ralph W. Powell, and E. R. Van Driest 1618

CONTENTS FOR DECEMBER, 1944 (*Continued*)

PAGE

Stability and Stiffness of Cellular Cofferdams.
By G. G. Greulich, and Raymond P. Pennoyer 1625

Pressure on the Lining of Circular Tunnels in Plastic Soils.
By John M. Bird 1639

INDEXES

Subject and Author Indexes, 1944 *Proceedings* 1649

A list of "Current Papers and Discussions" may be found on the page preceding the table of contents

The Society is not responsible for any statement made or opinion expressed in its publications

HINTS TO AUTHORS

Those who are planning papers for submission to PROCEEDINGS *will expedite Committee action measurably by first studying the standard instructions as to style, content, and format. For reprints, address the Secretary, Committee on Publications, 33 W. 39th St., New York 18, N. Y.*

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

FLOW IN A CHANNEL OF DEFINITE ROUGHNESS

BY RALPH W. POWELL,[1] M. AM. SOC. C. E.

SYNOPSIS

Although the effect of definite roughness of the wall upon the resistance to the flow of fluids in pipes has been thoroughly studied, there seems no equally satisfactory study of the effect of known wall roughness on the flow in open channels. This paper reports the results of some experiments bearing on this question which were performed by the writer during the summer of 1940 in the laboratory of the Institute of Hydraulic Research at Iowa City, Iowa. A total of two hundred runs on a rectangular flume, with eleven different roughnesses and four different slopes, indicated that Chézy's C could be expressed as $40 \log_{10} R/\epsilon + c_s$, in which R is the hydraulic radius, ϵ is a measure of the roughness, and c_s is a small shape correction. The mean of the deviations of the formula from the data was 2.1, and the mean percentage deviation was 4.4%.

In the same flume without roughness twenty-one runs of tranquil flow gave the formula $C = 40 \log_{10} \mathbf{R}/C + c_s + 4.8$, in which $\mathbf{R}$ is the Reynolds number. The mean deviation of this formula from the data was 2.32 or about 2.0% of the value of C. Shooting flow in the same smooth flume gave the formula $C = 60 \log_{10} \mathbf{R}/C + 30\,\mathbf{F} + c_s - 175$, in which $\mathbf{F}$ is the Froude number. The mean deviation of this formula from the data was 2.44, or about 2.1% of the value of C.

If these data are correct, two surprising results follow: (1) The "universal constant," $b = 2.5$, changes from 2.5 for pipes to about 3.0 in open channels; and (2) the law of resistance to shooting flow is quite different from the law of resistance to tranquil flow.

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by May 1, 1945.

[1] Associate Prof. of Mechanics, Ohio State Univ., Columbus, Ohio.

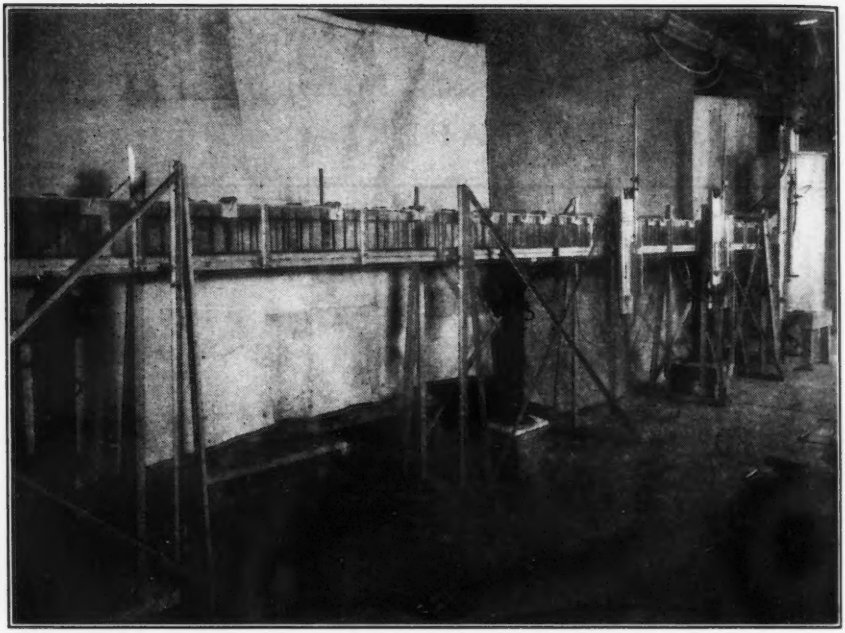


FIG. 1.—GENERAL VIEW OF APPARATUS, FACING UPSTREAM

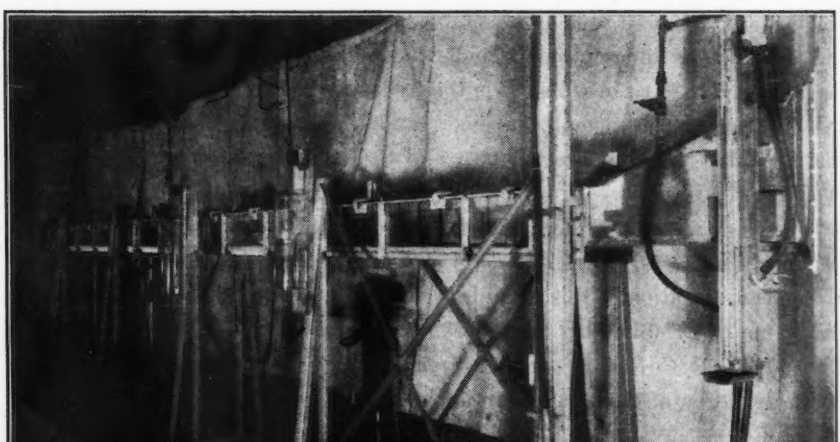


FIG. 2.—GENERAL VIEW OF APPARATUS, FACING DOWNSTREAM

Historical

The pioneer research on the flow of water in open channels with definite artificial roughness was made by H. Bazin[2,3] between 1855 and 1860. These tests were made in a channel 2 m wide and nearly 600 m long, and the measurements were quite accurate; but only a few of the roughnesses used were so definitely described that they could be accurately duplicated today, and apparently Bazin made no attempt to reduce his results to a formula that included a definite roughness figure. In fact, the idea of a definite linear unit of roughness seems not to have originated until the twentieth century.[4]

Measurements of the resistance to flow in pipes of definite roughness seem to have been first made by L. Hopf[5] and K. Fromm[6] in 1923. A rational theory of velocity distribution and, from it, formulas for resistance to flow were later developed by Theodor von Kármán,[7] M. Am. Soc. C. E., and Ludwig Prandtl.[8] In 1933, J. Nikuradse[9] published his epoch-making paper combining Prandtl's theory with definite experimental measurements of the resistance in pipes lined with sand grains of definite size. Since then a number of studies of flow in pipes of definite roughness have been made, of which may be mentioned those by V. L. Streeter,[10] Assoc. M. Am. Soc. C. E., H. Schlichting,[11] C. F. Colebrook[12,13] and C. M. White.[13]

The corresponding problem of the effect of definite roughness on flow in open channels does not seem to have received much attention. Mr. Keulegan[3] has published a good discussion of the problem, but with no experimental values other than Bazin's. The experiments described in this paper are a preliminary attempt to do for open channels what Nikuradse did for pipes.

Apparatus and Experimental Procedure

The flume used was 50 ft long, 8 in. wide, and 7 in. deep. The side-walls were of lucite and the base was a structural steel channel with its back upper-

[2] "Recherches hydrauliques," by H. Bazin, Paris, 1865. His results reduced to English units are given in "Flow of Water in Rivers and Other Channels," by E. Ganguillet and W. R. Kutter, translated into English by Rudolph Hering and John C. Trautwine, John Wiley & Sons, Inc., New York, N. Y., 1893. (The temperatures are not given in the translation, but they are evidently in the original as they are given by G. H. Keulegan.)

[3] "Laws of Turbulent Flow in Open Channels," by Garbis H. Keulegan, *Research Paper RP 1151*, National Bureau of Standards (U. S.), Vol. 21, December, 1938, pp. 707–741.

[4] "Elemente der technischen Hydromechanik," by Richard von Mises, Leipzig, 1914, p. 50, et seq.

[5] "Die Messung der hydraulischen Rauhigkeit," by L. Hopf, *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 3, 1923, pp. 329–339.

[6] "Strömungswiderstand in rauhen Rohren," by K. Fromm, *ibid.*, pp. 339–358.

[7] "Mechanische Ähnlichkeit und Turbulenz," by Theodor von Kármán, *Proceedings*, 3d International Cong. for Applied Mechanics, Stockholm, 1930, pp. 85–93.

[8] "Zur turbulenten Strömungen in Rohren und längs Platten," by Ludwig Prandtl, Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen, Vol. IV (1932), pp. 18–29; or "Neuere Ergebnisse der Turbulenzforschung," by L. Prandtl, *Zeitschrift des Vereines Deutscher Ingenieure*, Bd. 77, No. 5 (1933), pp. 105–114.

[9] "Strömungsgesetze in rauhen Rohren," by J. Nikuradse, *Forschungsarbeiten auf dem gebiete des ingenieur-wesens*, Heft 361, Berlin, 1933.

[10] "Frictional Resistance in Artificially Roughened Pipes," by Victor L. Streeter, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), pp. 681–713.

[11] "Experimentelle Untersuchungen zum Rauhigkeitsproblem," by H. Schlichting, *Ingenieur-Archiv*, Vol. VII, No. 1 (February, 1936), pp. 1–34. An abridged translation of the paper is given in *Proceedings*, Am. Soc. C. E., November, 1937, pp. 16–31 (following p. 1834).

[12] "Turbulent Flow in Pipes, with Particular Reference to the Transition Region between the Smooth and Rough Pipe Laws," by C. F. Colebrook, *Journal*, Inst. of Civ. Engrs., Vol. 11, 1939, pp. 133–156.

[13] "Experiments with Fluid Friction in Roughened Pipes," by C. F. Colebrook and C. M. White, *Proceedings*, Royal Soc. of London, Series A, Vol. 161, 1937, pp. 367–381.

most, painted with aluminum paint. The flanges of the channel offered attachment for the lucite side panels and for the vertical light steel angles which supported these panels at 20-in. intervals. The lucite panels were also stiffened along the upper edge by a lucite flange. A light steel strap was placed across the top at the middle of each panel and bolted to these flanges. At six points along its length the flume rested on blocks clamped to vertical trestles, as shown in Figs. 1 and 2.

The definite wall roughness was provided by square steel strips which extended down the sides and across the bottom so as to act as cleats or battens. The arrangements of the battens to create the various roughnesses are shown in Fig. 3, the larger of the two sizes being $\frac{1}{4}$ in. square and the smaller, $\frac{1}{8}$ in. square.

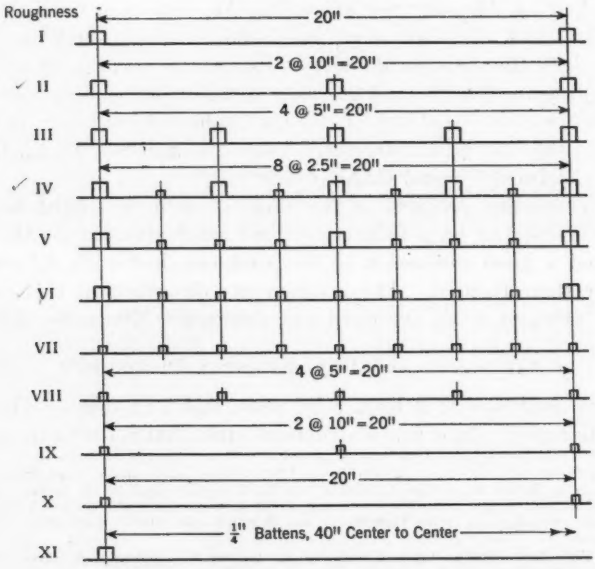


FIG. 3.—SPACING OF BATTENS TO FORM THE VARIOUS ROUGHNESSES

The water was measured by a diaphragm orifice in the 6-in. supply line just visible in the upper right corner of Fig. 1. For the larger flows the differential head on this orifice was measured by the mercury manometer shown at the right of Fig. 2. For most of the flows for which the differential head would have been less than 2 in. of mercury, it was read directly in inches of water in the vertical tubes shown to the left of the mercury manometer.

At the downstream end of the flume a control gate was arranged as shown in Fig. 4(*e*). This allowed part of the flow to pass under and part over, but by adjusting the wing nut (shown at the upper right) the depth at the downstream gage could be adjusted so that the flow throughout the flume was practically uniform. A thermometer is shown at the left in Fig. 4(*e*). The depth was read by hook gages in wells as shown at the right of Fig. 1 and the

left of Fig. 2. The term "second slope" in the caption of Fig. 4 means that the bottom slope was 0.0080. The wells were connected by rubber hose with the static pressure tubes shown in Figs. 4(*a*), 4(*c*), 4(*d*), 4(*f*), 5(*a*), and 6(*d*). These were made of copper tubing, 0.187 in. outside diameter, with the upstream end plugged with solder and filed to a "streamlined" contour. Four

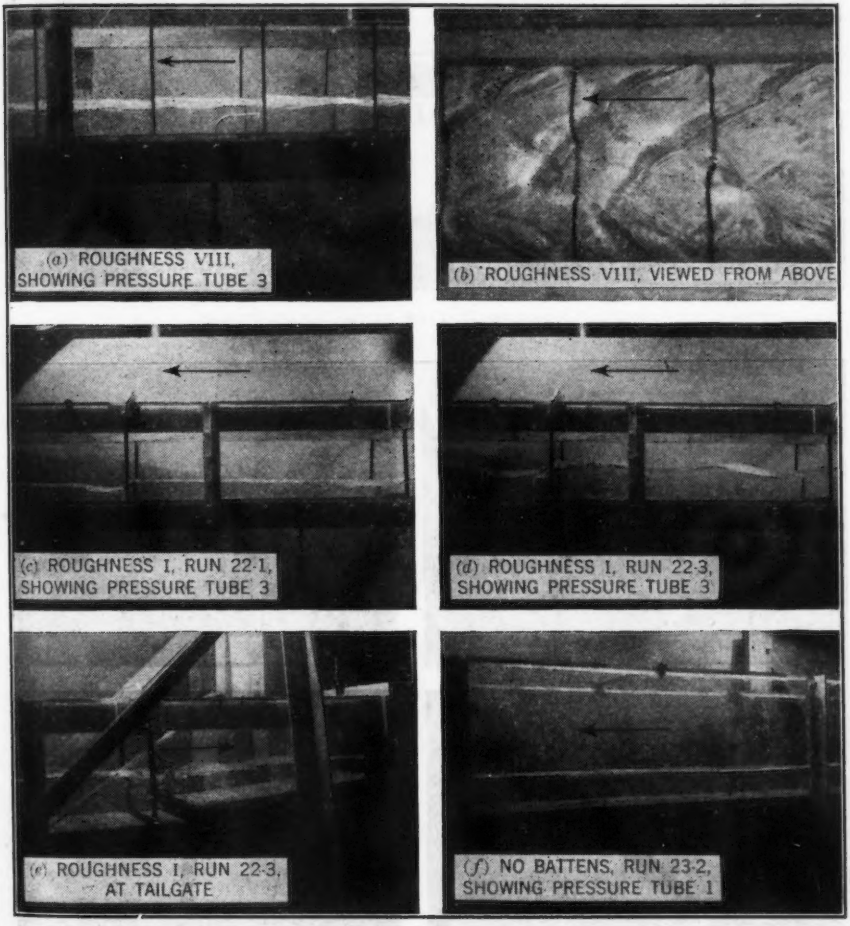

(a) ROUGHNESS VIII, SHOWING PRESSURE TUBE 3
(b) ROUGHNESS VIII, VIEWED FROM ABOVE
(c) ROUGHNESS I, RUN 22-1, SHOWING PRESSURE TUBE 3
(d) ROUGHNESS I, RUN 22-3, SHOWING PRESSURE TUBE 3
(e) ROUGHNESS I, RUN 22-3, AT TAILGATE
(f) NO BATTENS, RUN 23-2, SHOWING PRESSURE TUBE 1

FIG. 4.—EXAMPLES OF FLOW AT THE SECOND SLOPE

holes were drilled with a No. 54 drill, 1 in. from the nose—one on top, one on the bottom, and one on each side. It was felt that some such arrangement was necessary because the pressure at an ordinary flush peizometer opening would undoubtedly have been affected by the proximity of a batten, and, with flows such as those shown in Fig. 5(*a*) (which are similar to the flow in run 5–2), it would have been impossible to estimate the depth visually through the lucite

walls. The location of these pressure tubes is shown in Fig. 7. Further details regarding the apparatus and experimental procedure are given in the original draft of this paper on file in the Engineering Societies Library.[13a]

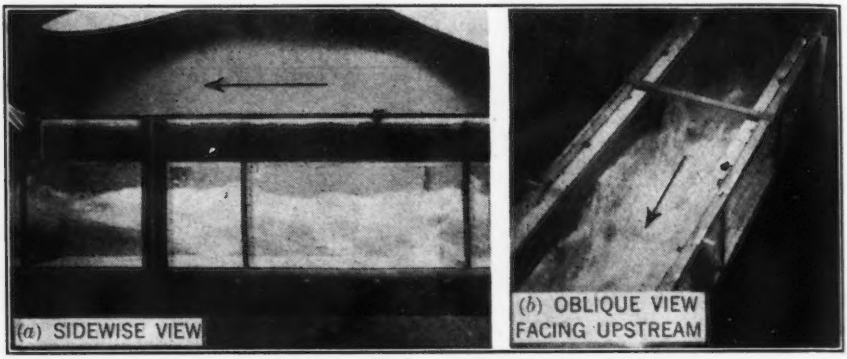


FIG. 5.—ROUGHNESS II, FLOW AT THE STEEPEST SLOPE

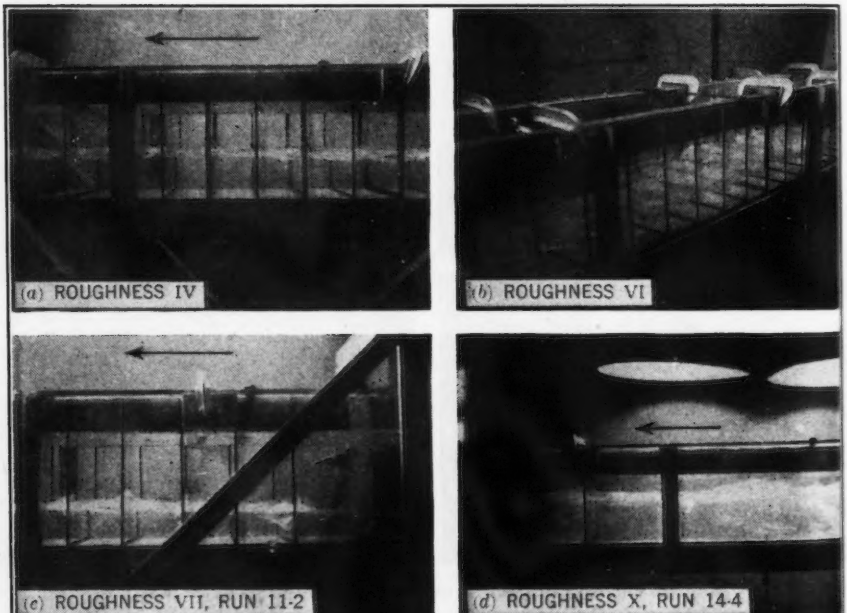


FIG. 6.—EXAMPLES OF FLOW AT THE STEEPEST SLOPE

When the work was begun the flume was set with its floor at a slope of $S = 0.0312$. This was steep enough to produce shooting flow in the smooth flume, and would not have been used to start with had it not already been

[13a] 29 West 39th St., New York, N. Y.

set up. After runs had been made at this slope with the smooth flume and with each of ten roughnesses (with about five different depths for each roughness), the slope was changed successively to 0.0080, 0.0020, and 0.0005, and similar runs were made at each slope. At the flattest slope many of the flows were too small to be measured by the orifice meter, even with the water-air manometer. For these a deflector was arranged at the downstream end to

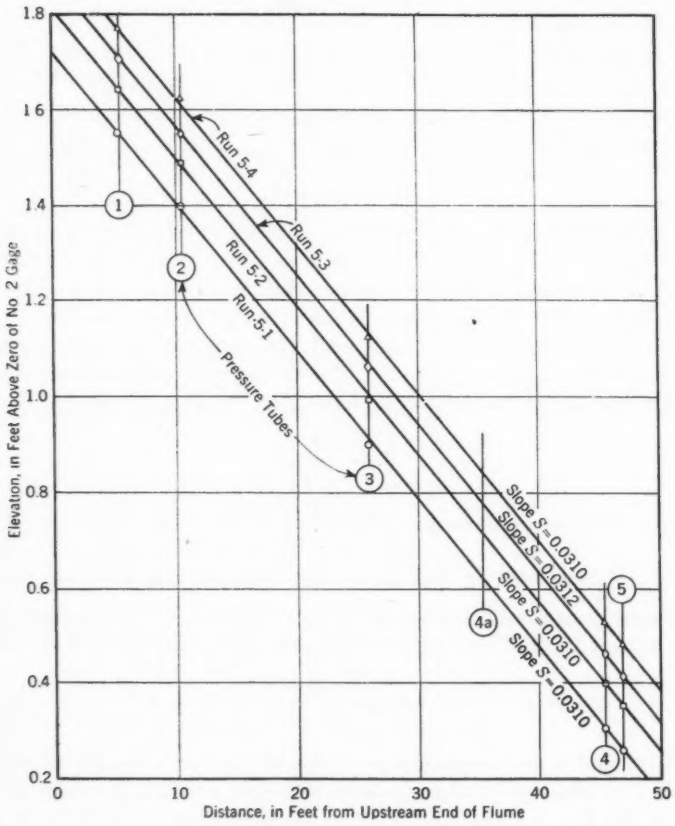


FIG. 7.—SAMPLE OF ENERGY GRADIENTS

direct the flow into a weighing tank. The time required to run 300 lb of water was measured with a stop-watch, except for a few of the smallest flows where the quantity was reduced to 200 lb, or even only 100 lb; but the time for these smaller weights was never less than 64 sec and was read to fifths of a second, so that these flows were probably determined more accurately than those measured with the orifice.

Before any runs were taken, the width of the channel was measured at ninety different sections and at two depths in each section with a micrometer gage reading to thousandths of an inch. The width at about 0.3 in. from the

floor varied from 8.148 in. to 8.215 in. with a mean of 8.193 ± 0.013 in. At a height of about 4 in. above the floor the width varied from 8.114 in. to 8.210 in. with a mean of 8.166 in. In general, each 20-in. panel was sprung outward so that the channel was about 0.03 in. wider at the middle of the panel than at the junction with adjoining panels; and these junctions sometimes involved an offset of 0.01 in. to 0.02 in. Therefore, the so-called "smooth channel" was not absolutely smooth but had a slight amount of roughness and an appreciable waviness.

The orifice meter had been calibrated a few months before these experiments by a student. The result of this calibration was put into the form of a curve of values of $Q/\sqrt{H}$ plotted against $\sqrt{H}$ to such a scale that the constant could be read to four significant figures.

Computations

The letter symbols in this paper are defined where they first appear and are assembled for reference in the Appendix. The method of computation is described in detail in the original draft of this paper on file in the Engineering Societies Library.[13a] The most important data and results are given in Table 1 (Appendix). The slope was determined from plotted energy gradients, a sample of which is shown in Fig. 7. For the steeper slopes there seemed the possibility that the resistance might depend on the Froude number,

$$\mathbf{F} = \frac{V}{\sqrt{g\,y}} \qquad (1)$$

Therefore, this was computed in addition to the usual R, V, C, and $\mathbf{R}$.

Analysis of Results with Smooth Channel

The most fundamental treatment of the resistance to flow in open channels seems to be that of Mr. Keulegan,[14] whose formula for smooth walled channels is

$$\frac{V}{V_f} = a_s - b + b \log_e \left(\frac{R\,V_f}{\nu}\right) + b\,\beta - \bar{\epsilon}\,\frac{V}{V_f} \qquad (2)$$

in which: a_s and b are constants; ν is the kinematic viscosity of the liquid flowing; β is a function of the shape of the cross section; $\bar{\epsilon}$ measures the resistance of the free surface; and V_f, the so-called friction velocity, is defined as follows:

$$V_f = \sqrt{\frac{\tau_o}{\rho}} = \sqrt{R\,S\,g} = \frac{\sqrt{g}\,V}{C} \qquad (3)$$

Eq. 2 can be written

$$C = \frac{\sqrt{g}}{1 + \bar{\epsilon}}\left(a_s - b + b\,\beta + b \log_e \frac{\sqrt{g}}{4} + b \log_e \frac{\mathbf{R}}{C}\right) \qquad (4)$$

in which $\mathbf{R} = 4\,R\,V/\nu$ = the Reynolds number. Mr. Keulegan shows that

[14] "Laws of Turbulent Flow in Open Channels," by Garbis H. Keulegan, *Research Paper RP 1151*, National Bureau of Standards (U. S.), Vol. 21, December, 1938, Eq. 48.

for a rectangular channel

$$\beta = \log_e \left(1 + \frac{2y}{B}\right) - \frac{y}{B} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which y is the depth of the flow and B is the width. Assuming that b here has the same value as that which Nikuradse[9] found for pipes, and that $\bar{\epsilon}$ is zero, the shape correction term is equal to

$$c_s = \frac{b\beta\sqrt{g}}{1+\bar{\epsilon}} = 14.18\beta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Values of this term were calculated for various depths and are plotted against the corresponding hydraulic radius in Fig. 8. Since this term represents such a small part of the total C, it is felt that the approximation of making $\bar{\epsilon} = 0$ and $b = 2.5$ will introduce no appreciable error. By grouping the remaining terms, Eq. 4 may be rewritten as

$$C - c_s = m \log_{10} \frac{\mathbf{R}}{C} + J \quad \ldots\ldots (7)$$

in which the logarithm is now to the base 10, and

$$m = \frac{2.3026\, b\sqrt{g}}{1+\bar{\epsilon}} \quad \ldots\ldots\ldots (8a)$$

The same assumptions as those in Eq. 6 would make $m = 32.65$. Define J as:

$$J = \frac{\sqrt{g}}{1+\bar{\epsilon}} \left(a_s - b + b \log_e \frac{\sqrt{g}}{4}\right) \ldots (8b)$$

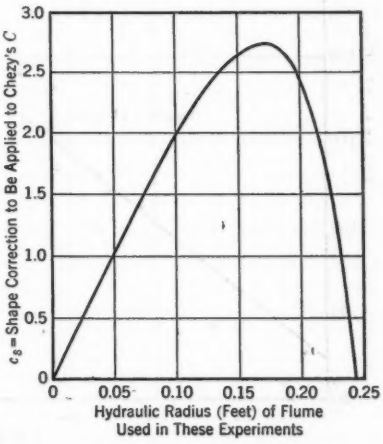


FIG. 8.—SHAPE CORRECTION

Using Nikuradse's value of $a_s = 5.5$ in addition to these assumptions, J would be $\sqrt{32.16}\,(5.5 - 2.5 + 2.5 \log_e \sqrt{2.01}) = 21.96$ and

$$C - c_s = 32.65 \log_{10} \frac{\mathbf{R}}{C} + 21.96 \quad \ldots\ldots\ldots\ldots\ldots (9a)$$

It may be noted that for a semicircular cross section, Mr. Keulegan shows that $\beta = 0.259$, so that $c_s = 14.18 \times 0.259 = 3.67$, and

$$C = 32.65 \log_{10} \frac{\mathbf{R}}{C} + 25.63 \quad \ldots\ldots\ldots\ldots\ldots (9b)$$

which is almost identical with the formula for flow in smooth channels derived independently by the writer[15] in 1938.

However, when $C - c_s$ was plotted against $\log_{10} \mathbf{R}/C$, the data for this smooth channel (Fig. 9) failed to agree with Eq. 9a. In the first place the points for

[15] "Mechanics of Liquids," by Ralph W. Powell, Macmillan Co., New York, N. Y., 1940, p. 202.

the steepest slope, where the flow was definitely shooting (with **F** = 2.49 or more), fall in a distinct region indicating an entirely different law. The others lie fairly well along a straight line, but its equation, fitted by least squares, is

$$C - c_s = 39.7 \log_{10} \frac{\mathbf{R}}{C} + 5.6 \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

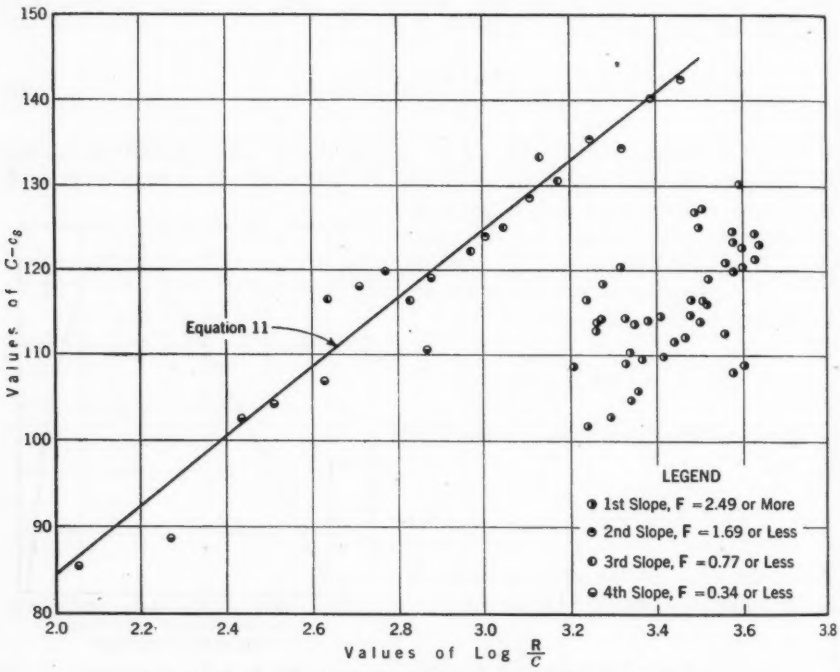


FIG. 9.—VALUES OF CHÉZY COEFFICIENT C FOR A SMOOTH CHANNEL

The probable error of m, also computed by least squares, is 1.4, so that it was decided to replace 39.7 ± 1.4 by 40. This changes the most probable value of J from 5.6 to 4.8, so that Eq. 10 becomes

$$C - c_s = 40 \log_{10} \frac{\mathbf{R}}{C} + 4.8 \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

This line is drawn in Fig. 9. Table 1(*a*) in the Appendix gives the values of C computed by Eq. 11 (with the help of a table of values of $C + 40 \log_{10} C$), and also the deviations from the observed C. The mean deviation was 2.32 and the "probable error" was 2.0. Since C averaged 117.4, the average error was about 2.0% and the probable error about 1.7%.

If the walls are wavy, the value of J in Eq. 7 would be expected to be less than that for straight walls, but the value of m would be expected to remain the same. However, the value of m found in these experiments is definitely more than that found by Nikuradse. The value of m = 39.7 ± 1.4 corre-

sponds to $b = 3.04 \pm 0.11$ if $\bar{\epsilon} = 0$ and even more if $\bar{\epsilon}$ has an appreciable size. Nikuradse's value of $b = 2.5$ is probably somewhat approximate (notwithstanding the numerous formulas that have been based on it), but it can scarcely be this much in error. Therefore, there are either serious systematic errors in these experiments, or b has a different value for channels than for pipes. The much more extensive tests on rough channels, reported in this paper, also indicated this same value of m. It may also be noted that the study of velocity distribution in open channels by E. H. Taylor,[16] Jun. Am. Soc. C. E., following a modification of a method proposed by M. P. O'Brien,[17] M. Am. Soc. C. E., gave an average value for the "universal constant" of 0.306, which corresponds to $b = 3.27$ and which is even more than the writer's experiments show.

As stated, Eq. 11 does not represent the runs at the steepest slope (setups 1, 2, 3, and 8, in Table 1(*b*) in the Appendix). When the discrepancies from Eq. 11 were plotted against the Froude number **F**, there was no apparent relationship for the flatter slopes (setups 23, 24, and 46, in Table 1(*a*), in which **F** varied from 0.26 to 1.69); but on the steepest slope where **F** varied from 2.49 to 3.28, there was a very definite dependence on **F**, which it seemed might be well expressed by a formula of the type

$$C - c_s = J + z \log_{10} \frac{\mathbf{R}}{C} + x\,\mathbf{F} \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Solving for the most probable values of J, z, and x by least squares gave: $x = 30.4 \pm 2.2$; $z = 58.7 \pm 3.1$; and $J = -171.5$. In view of the fairly large probable errors it seemed best to round these values to $x = 30$ and $z = 60$, which for the best fit necessitated changing J to -175 and gave the formula

$$C - c_s = 30\,\mathbf{F} + 60 \log_{10} \frac{\mathbf{R}}{C} - 175 \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

The values of C computed by Eq. 13 (with the aid of a table of values of $C + 60 \log_{10} C$) and the deviations from the observed C are given in Table 1(*b*) in the Appendix. The mean deviation was 2.44 and the probable error 2.1. Since C averaged 117.9, this makes the average error about 2.1% and the probable error about 1.8%.

Eq. 13 must be regarded as tentative until checked by other investigators and at other slopes. When it is realized how wavy the water surface was in this case (no photograph was taken, but the surface was similar to that in Fig. 4(*d*), it is not surprising that **F** enters into this case although it did not enter when the flow was more tranquil. It is surprising, however, that the coefficient of $\log_{10} \mathbf{R}/C$ should change. It is also surprising that the change from Eq. 11 to Eq. 13 came between $\mathbf{F} = 1.69$ and $\mathbf{F} = 2.49$ instead of at $\mathbf{F} = 1.00$. (Tests at one or more intermediate slopes would be required to determine where the change occurs.) The six runs of setup 23, Table 1(*a*) (Appendix), had Froude numbers **F** varying from 1.45 to 1.69 and agreed

[16] "Velocity-Distribution in Open Channels," by E. H. Taylor, *Transactions*, Am. Geophysical Union, 1939, pp. 641–643.

[17] "The Vertical Distribution of Velocity in Wide Rivers," by Morrough P. O'Brien, *ibid.*, 1937, pp. 467–470.

with Eq. 11 with an average discrepancy of 0.87, whereas the average discrepancy from Eq. 13 was 53.4.

Analysis of Results with Rough Channel

Mr. Keulegan's formula for rough walled channels, in the notation of this paper, is

$$\frac{V}{V_f} = a_r - b + b \log_e \left(\frac{R}{\epsilon}\right) + b\beta - \bar{\epsilon}\frac{V}{V_f} \quad \ldots\ldots\ldots\ldots (14a)$$

in which a_r, a constant for rough walls, takes the place of the a_s in Eq. 2, and ϵ is a measure of the roughness of the channel walls (which Messrs. Nikuradse and Keulegan represented by k). Substituting the value of V_f and rearranging Eq. 14*a*,

$$C = \frac{\sqrt{g}}{1 + \bar{\epsilon}}\left[a_r - b + b\beta + b \log_e \left(\frac{R}{\epsilon}\right)\right] \quad \ldots\ldots\ldots\ldots (14b)$$

As in the case of smooth channels (see Eq. 6), $\frac{b\beta\sqrt{g}}{1 + \bar{\epsilon}}$ can be replaced by c_s and the other terms combined to make

$$C - c_s = J + m \log_{10} \frac{R}{\epsilon} \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

For roughness I, values of $C - c_s$ were plotted against $\log_{10} R$. Since roughness IX (Fig. 3) constituted a half-scale model of roughness I except for width (which was corrected for in each case by the c_s-term), the values for roughness IX, with hydraulic radii twice those observed, were also plotted on the same sheet. Similarly, roughness VIII was combined with roughness II, roughness VII with roughness III, and roughness X with roughness XI. In each case the slope of the line $m = 40$ seemed to give as good a fit as any. This value was obtained visually without a least squares solution.

For roughness III, the equation thus obtained was

$$C - c_s = 72.2 + 40 \log_{10} R \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

If ϵ were taken as 0.25 in. = 0.02083 ft, $1/\epsilon = 48$, and $40 \log_{10} 1/\epsilon = 67.25$, J would be $72.2 - 67.25 = 4.95$. On the other hand, $4.95/40 = 0.12375 = \log_{10} 1.327$ and the reciprocal of 1.327 is 0.754, so that if ϵ had been taken as about three quarters the actual height, J would have been zero. Since the measure of roughness depends not only upon the height of the particle, but also on the spacing, it is obvious that the unit of measurement of roughness is quite arbitrary, and that it would be quite allowable to take it such that $J = 0$. Therefore, the following formula was used:

$$C - c_s = 40 \log_{10} \frac{R}{\epsilon} = 40 \log_{10} R - 40 \log_{10} \epsilon \quad \ldots\ldots\ldots\ldots (17)$$

From the aforementioned plottings, and similar ones for roughnesses IV, V, and VI, ϵ was obtained and then C was computed for each run and the discrepancy between the computed and observed values of C was arranged in

Table 1(*c*) to Table 1(*m*) in the Appendix. If there was any serious difference between the sums of the plus and minus discrepancies, the value of ϵ was adjusted to make the discrepancies balance. The final values are listed in Table 2(*a*) in the Appendix. Table 2 also gives the values of ϵ/h, s, and s/h, in which h is the actual dimension of the batten, and s is the spacing of the battens center to center. Since roughnesses IV, V, and VI consisted of both $\frac{1}{8}$-in. and $\frac{1}{4}$-in. battens, no value of h was assigned to them.

Probable Accuracy

According to probability theory, the percentage probable error of the product or quotient of two observed quantities is the square root of the sum of the squares of the percentage probable errors of the factors. Estimates were made of the probable error in each of the various kinds of readings, and, from them, of the probable errors in the computed quantities, based on the aforementioned probability theory. In the case of the slope, the deviation of the plotted points from a straight profile also gave an indication of the probable error. These estimates gave percentage probable errors, as follows: For the velocity, 1.5%; for the slope, 1.7%; for the hydraulic radius, 1.3%; and for C, 1.9%. These are only estimates and may be too low. At any rate the average deviation of the 200 runs on rough channels from Eq. 17 was 4.37%, which by C. A. F. Peters' formula (probable error is approximately 0.845 times the average deviation) would indicate a probable error of 3.7% in one observation. This includes any error due to the choice of a wrong general form for Eq. 17, any error in the value $m = 40$, and any error due to the wrong choice of ϵ for each particular roughness. As to the value of m for tranquil flow in the smooth channel, it already has been noted that the least squares solution gave a probable value of 39.7 ± 1.4, and as the average value of $\log_{10} \mathbf{R}/C$ was 3.441, the probable error in the term $40 \log_{10} \mathbf{R}/C$ might be estimated at $1.4 \times 3.441 = 4.8$; but the error in the computed value of C would not be this much as it is largely compensated for by choosing a value of J to correspond to the m used. No study was made of the accuracy of the value $m = 40$ in Eq. 17, but from the plotting the writer would judge the probable error to be about 1.5.

The shape correction, c_s, is of the same order of magnitude as the discrepancies, so that the data are insufficient to check its correctness; but a plotting of the discrepancies against values of the depth y showed no systematic relationship, so that there is no evidence against its correctness. It may be mentioned again that the relation $c_s = 14.18\beta$ was based on Nikuradse's value of b. The writer's value of $m = 40$, instead of 32.65, is an increase of 22.5% and would increase the coefficient of β from 14.18 to 17.37. On the other hand, $\bar{\epsilon}$, the free surface effect, was taken as zero. If it has an appreciable value, part or all of the difference would be compensated for. At any rate the maximum change in c_s could be only 0.6, so that it was thought unnecessary to recompute all the data.

In general, it must be admitted that the accuracy of the experiments recorded herein is only of about the same order as those of Osborne Reynolds[18]

[18] "An Experimental Investigation of the Circumstances Which Determine Whether the Motion of Water Shall Be Direct or Sinuous, and of the Law of Resistance in Parallel Channels," by Osborne Reynolds, *Philosophical Transactions*, Royal Soc. of London, Vol. 174, Pt. III (1883), p. 935.

on pipes in the 1880's. It can only be hoped that, in this case too, they may lead to more accurate measurements by other observers which will clear up the uncertainties.

It might be urged that, since quite large standing waves formed on the surface at the higher velocities in the rough channel, a term involving the Froude number should be included in Eq. 17. However, a plotting of the discrepancies from Eq. 17 against **F** showed no systematic relationship. This was to be expected because, since $\mathbf{F} = V/\sqrt{g\,y}$ and $C = V/\sqrt{R\,S}$, and since R varies nearly as y, resistance depending on **F** could hardly be distinguished from other resistance except by the difference in slopes, and in these tests there was only one slope at which **F** was large enough to have much effect.

Comparison with Bazin's Results

Runs 12 to 17 of Bazin's tests[2] were on rectangular plank channels with wood strips 1 cm thick and 2.7 cm wide nailed crosswise on the bottom and sides. His results, therefore, should be comparable to those herein presented except that the battens were not square. Since it seems that this might make relatively little difference, Table 1(*n*) (Appendix), has been computed. The value of ϵ which gave the best fit in Eq. 17 was selected (and listed in Table 2(*b*) in the Appendix), and the computed C and the deviations from the observed C are given in Table 1(*n*). The mean discrepancy was only 1.6, as compared to 2.1 for the writer's data.

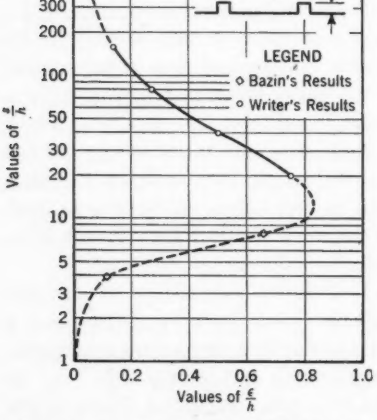


Fig. 10.—Relation of Relative Roughness to Spacing

The ratios of the equivalent roughness to the actual height of the "roughness particle" and the ratio of spacing to height, as given in Table 2 in the Appendix, are plotted in Fig. 10. The dashed portion of this curve is admittedly uncertain, but it indicates that the spacing of square battens that would give the maximum resistance is about twelve times the height of the battens, and that the maximum "equivalent roughness" is about 0.83 of the height. It must be remembered that the scale of measurement of "equivalent roughness" was taken arbitrarily to make J in Eq. 15 equal to zero, and that therefore this last result would not necessarily agree with Nikuradse's sand grain size and his formula for pipes. The facts that Schlichting[11] obtained a value of $\epsilon/h = 11.9$ for the flanges of a small angle section placed across his pipe, and that Mr. Streeter[10] found values of ϵ/h as high as 4.57 for square circumferential grooves, both investigators using ϵ equal to Nikuradse's k, indicate that the unit of ϵ used in Eq. 17 may be, say, from five to ten times as large as that used by Nikuradse. Although a standard

value would be desirable, it would not seem feasible to adopt one until some one geometrical type of roughness (such as the one used in this study or some other that could be reproduced as easily) has been carefully calibrated by the Schlichting method in a pipe, and by some method comparable to that of the writer, in open channels. Then all other roughnesses could be referred to this scale.

Conclusions

The discrepancies of Eq. 17 from the observed data average nearly twice as much as would be expected from the probable inaccuracies of the measurements as estimated by the observer; but these are only estimates and it may be that Eq. 17 is entirely correct and the discrepancies entirely due to errors in observation or to errors of observation plus errors in the chosen values of ϵ. It seems more probable that the discrepancies are partly due to these two factors and partly due to wrong values in the constants of Eq. 17, but that the form of the equation is satisfactory, at least for tranquil flow.

For the case of the depth half the width, c_s is 2.74. The resulting formulas for this case are:

For tranquil smooth channels,

$$C = 7.54 + 40 \log_{10} \frac{\mathbf{R}}{C} \quad \ldots\ldots\ldots\ldots\ldots\ldots (18a)$$

and, for rough channels,

$$C = 2.74 + 40 \log_{10} \frac{R}{\epsilon} \quad \ldots\ldots\ldots\ldots\ldots\ldots (18b)$$

For laminar flow, assuming that the slope of the hydraulic gradient is the same as for a square pipe flowing full[19]

$$C = \sqrt{\frac{8\,g\,\mathbf{R}}{57.0}} = 2.12\,\sqrt{\mathbf{R}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (18c)$$

It should be noted that Eqs. 18*a*, 18*b*, and 18*c* apply only when the depth is half the width. For smaller depths the constant in Eq. 18*c* is less, approaching the value

$$C = \sqrt{\frac{8\,g\,\mathbf{R}}{96}} = 1.64\,\sqrt{\mathbf{R}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (18d)$$

when the width is infinite compared with the depth. Eqs. 18 are plotted in Fig. 11, which sums up most of the results of this paper. It must be understood that, as in the corresponding case for pipes, the same channel may act as rough or smooth. For example, a channel 4 ft wide flowing 2 ft deep, with an absolute roughness (ϵ) of 0.0010 ft, would have $R = 1.0$ ft and $R/\epsilon = 1{,}000$, and C would be 122.7 for any Reynolds number greater than 92,900; but, for Reynolds numbers less than this, it would behave as a smooth channel with C decreasing as $\mathbf{R}$ decreased down to about 67 at $\mathbf{R} = 2{,}000$, when Eq. 18*c*

[19] "A Review of Flow in Pipes and Channels," by S. J. Davis and C. M. White, *Engineering*, Vol. 128 (1929), p. 71. This simply puts into English the results of J. Boussinesq, *Journal de Mathématiques pures et appliquées de M. Liouville* (1868), p. 393.

would begin to apply. Of course, the region from $\mathbf{R} = 2{,}000$ to $\mathbf{R} = 8{,}000$ is uncertain. It has been studied by L. Hopf,[20] H. Jeffreys,[21] R. E. Horton, M. Am. Soc. C. E., the late H. R. Leach, Assoc. M. Am. Soc. C. E., and R. Van Vliet,[22] and by L. G. Straub,[23] M. Am. Soc. C. E. Professor Straub, however, used a different definition for Reynolds number and also seems to have used the constant for a round pipe in his equation for laminar flow.

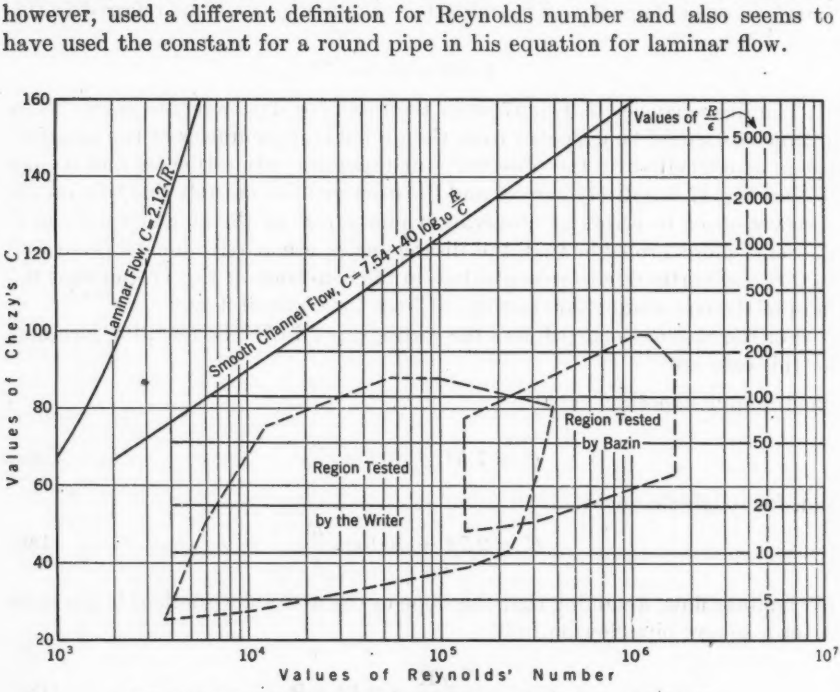


FIG. 11.—RELATION BETWEEN C AND R FOR RECTANGULAR CHANNELS WITH THE DEPTH EQUAL TO HALF THE WIDTH

By analogy with pipes one may suppose also that the region just to the right of the smooth channel line is probably not given correctly by the lines as drawn. One would expect that for uniform roughness there would be an upward bend (just as Nikuradse[9] found a downward bend for the friction coefficient f in pipes) and that for irregular roughnesses the bend would be in the opposite direction, as found by Messrs. Colebrook and White.[13] This transition range in pipes has been discussed quite fully by Mr. Colebrook[12] and by Hunter Rouse,[24] M. Am. Soc. C. E. Unfortunately, the writer's tests did not include any case in which the rough channel acted as smooth. It

[20] "Turbulenz bei einem Flusse," by L. Hopf, *Annalen der Physik* (4) xxxii (1910), pp. 777–808.

[21] "The Flow of Water in an Inclined Channel of Rectangular Section," by Harold Jeffreys, *Philosophical Magazine and Journal of Science*, London, Edinburgh, and Dublin, Vol. 49 (May, 1925), pp. 793–807.

[22] "Laminar Sheet-Flow," by Robert E. Horton, H. R. Leach, and R. Van Vliet, *Transactions*, Am. Geophysical Union, 1934, pp. 393–404.

[23] "Studies of the Transition Region Between Laminar and Turbulent Flow in Open Channels," by Lorenz G. Straub, *ibid.*, 1939, pp. 649–653.

[24] "Evaluation of Boundary Roughness," by Hunter Rouse, *Proceedings*, 2d Hydraulic Conference, Iowa City, Iowa, 1943, pp. 105–116.

would be a difficult condition to arrange for water, but very easy with a liquid of the right viscosity.

In all the computations of this paper the area and hydraulic radius are based on the gross section of the channel without the battens. If the battens were quite close together there certainly would be a question as to whether computation should not be based on the net section, or on some kind of average section. This knotty question is left for further study.

If Fig. 11 correctly represents the flow of liquids in channels, the implications for river model studies are evident. If the prototype acts as a rough channel, it is not sufficient to give the model the same relative roughness, because, at its smaller Reynolds number, it might be acting as a smooth channel. The only solution seems to be to use a sufficiently large model to bring it into the rough channel range.

Acknowledgments

The writer wishes to express his grateful appreciation of the generosity of the Iowa Institute of Hydraulic Research which placed its facilities at his disposal, and met the incidental expenses of the study; especially to F. M. Dawson, M. Am. Soc. C. E., Director of the Institute, E. W. Lane, M. Am. Soc. C. E., then Associate Director in Charge, and to Hunter Rouse, M. Am. Soc. C. E., and A. A. Kalinske, Assoc. M. Am. Soc. C. E., and to A. M. McLeod. He also wishes to thank Paul Nemenyi and James Robertson, Jun. Am. Soc. C. E., at that time both connected with the Institute, and to express his appreciation of the work of E. W. Thomas and Dale Harris, technicians, who assisted in the construction of the apparatus. Several students, working under provisions of the National Youth Administration, assisted with the computations.

APPENDIX

Notation and Tables of Observational Data

The following symbols, used in the paper, conform essentially to American Standard Letter Symbols for Hydraulics,[25] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1942:

a = a constant used by Mr. Keulegan,[3] subscript r denoting rough channels and subscript s denoting smooth channels;
B = channel width;
b = a constant, the reciprocal of von Kármán's "universal constant";
C = the Chézy coefficient in the formula $V = C\sqrt{RS}$;
c = a correction; c_s = a small shape correction;
$\mathbf{F}$ = Froude number = $V/\sqrt{g\,y}$;
f = coefficient of pipe friction in the formula $S = \dfrac{f\,V^2}{D\,2\,g}$;
g = acceleration of gravity;
H = head on the measuring orifice;
h = height of roughness particles, or battens;
J = a constant defined by Eq. 8*b*;
k = a symbol for roughness used by Messrs. Nikuradse and Keulegan (see ϵ);
m = a constant in Eq. 7, defined by Eq. 8*a*;
Q = rate of flow;
R = hydraulic radius;
$\mathbf{R}$ = Reynolds number;
S = slope of the energy gradient;
s = spacing of roughness battens (center to center);
V = mean velocity in the cross section of a flume or channel; V_f = friction velocity as defined by Eq. 3;
x = the unknown coefficient of $\mathbf{F}$ in Eq. 12;
y = depth of flow; that is, vertical measurements or measurements parallel to the Y-axis;
z = an unknown coefficient in Eq. 12;
β = a function of the shape of the cross section as defined by Eq. 5;
ϵ = absolute or equivalent roughness; that is, a measure of the roughness of the channel walls (see k);
$\bar{\epsilon}$ = a measure of the resistance of the free surface;
ν = kinematic viscosity of the flowing liquid;
ρ = density of the flowing liquid;
τ = shearing stress; τ_o = shearing stress at the liquid boundary.

[25] ASA—Z10.2—1942.

TABLE 1.—OBSERVED TEST DATA AND COMPUTED RESULTS

Run[a]	y (ft)	R (ft)	V (ft per sec)	S $\times 10^4$	Observed C	F	t (degrees Centigrade)	$\mathbf{R}e$ $\times 10^{-4}$	c_s	Computed C	Discrepancy −	Discrepancy +
(*a*) SMOOTH CHANNEL; TRANQUIL FLOW; DISCREPANCIES BY EQ. 11												
46-1	0.076	0.062	0.407	2.9	86.9	0.26	22.1	0.975	1.3	87.9	....	1.0
46-2	0.103	0.079	0.548	3.8	90.5	0.30	21.9	1.67	1.6	96.2	....	5.7
46-3	0.148	0.103	0.708	4.3	104.6	0.32	22.0	2.82	2.0	104.1	0.5	....
46-4	0.187	0.121	0.727	2.9	106.4	0.30	22.2	3.40	2.3	107.2	....	0.8
46-5	0.239	0.140	0.941	4.3	119.8	0.34	22.1	5.11	2.6	113.6	6.2	....
46-6	0.298	0.159	0.999	4.8	120.9	0.32	22.1	6.14	2.7	116.4	4.5	....
46-7	0.349	0.172	1.080	4.8	122.7	0.32	22.1	7.20	2.7	118.8	3.9	....
46-8	0.410	0.186	1.149	5.3	113.2	0.32	22.1	8.26	2.7	120.9	....	7.7
46-9	0.473	0.198	1.183	5.3	121.7	0.30	22.1	9.05	2.6	122.2	....	0.5
24-1	0.109	0.083	1.417	12	118.2	0.76	27.1	5.08	1.7	112.6	5.6	....
24-2	0.159	0.108	1.667	17	118.6	0.74	27.1	7.85	2.1	119.6	....	1.0
24-3	0.217	0.133	1.990	20	124.7	0.75	27.1	11.5	2.5	125.7	....	1.0
24-4	0.270	0.151	2.16	20	127.6	0.73	27.1	14.1	2.6	129.0	....	1.4
24-5	0.325	0.166	2.51	26	136.1	0.77	27.1	18.1	2.7	132.9	3.2	....
24-6	0.404	0.185	2.55	23	133.3	0.71	24.7	19.4	2.7	133.9	....	0.6
23-1	0.118	0.088	3.30	76	125.8	1.69	27.3	12.6	1.8	126.5	....	0.7
23-2	0.150	0.104	3.68	74	130.6	1.67	27.2	16.6	2.0	130.9	....	0.3
23-3	0.214	0.131	4.22	75	137.7	1.61	27.2	24.1	2.4	137.0	0.7	....
23-4	0.256	0.146	4.50	74	137.0	1.57	27.2	28.6	2.6	139.9	....	2.9
23-5	0.317	0.164	4.91	75	143.1	1.54	27.1	35.0	2.7	143.1	0.0	....
23-6	0.397	0.183	5.18	77	145.0	1.45	27.1	41.2	2.7	145.6	....	0.6
(*b*) SMOOTH CHANNEL; SHOOTING FLOW; DISCREPANCIES BY EQ. 13												
1-1	0.114	0.085	5.17	293	103.4	2.70	23.6	17.7	1.7	102.1	1.3	....
1-2	0.140	0.099	5.89	281	111.3	2.77	23.7	23.5	2.0	109.9	1.4	....
1-3	0.149	0.104	6.16	274	115.6	2.81	23.9	25.8	2.0	112.8	2.8	....
1-4	0.224	0.135	7.02	258	118.7	2.62	24.0	38.4	2.5	117.0	1.7	....
1-5	0.257	0.146	7.50	252	123.5	2.61	24.0	44.4	2.6	120.0	3.5	....
2-4	0.111	0.084	5.81	288	118.3	3.07	25.4	20.3	1.7	113.9	4.4	....
2-6	0.115	0.086	5.75	293	114.5	2.99	25.4	20.7	1.7	112.4	2.1	....
2-3	0.116	0.086	5.82	293	115.6	3.01	25.1	20.9	1.7	113.0	2.6	....
2-2	0.118	0.088	5.95	299	116.1	3.05	25.1	21.6	1.8	114.7	1.4	....
2-5	0.119	0.088	6.11	294	120.1	3.12	25.4	22.5	1.8	117.3	2.8	....
2-1	0.125	0.091	6.58	318	122.1	3.28	25.1	25.0	1.8	123.6	....	1.5
2-11	0.130	0.094	6.03	286	116.3	2.95	27.1	24.6	1.9	115.2	1.1	....
2-12	0.143	0.101	6.45	306	116.2	3.01	27.0	28.2	2.0	119.6	....	3.4
2-13	0.141	0.100	6.36	300	116.2	2.98	27.0	27.5	2.0	118.3	....	2.1
2-7	0.197	0.125	7.63	280	129.1	3.03	25.4	39.8	2.4	127.9	1.2	....
2-8	0.199	0.126	7.62	285	127.4	3.01	25.5	40.1	2.4	127.6	....	0.2
2-9	0.200	0.126	7.65	287	127.1	3.02	25.5	40.4	2.4	128.0	....	0.9
2-10	0.271	0.151	8.24	255	132.9	2.79	25.6	52.2	2.6	128.0	4.9	....
3-1	0.100	0.077	5.22	291	110.3	2.91	27.4	17.6	1.6	107.0	3.3	....
3-2	0.130	0.094	5.95	298	112.4	2.91	27.2	24.4	1.9	114.0	....	1.6
3-3	0.137	0.098	6.12	308	111.6	2.92	27.3	26.1	1.9	115.7	....	4.1
3-4	0.156	0.107	6.40	281	116.7	2.86	27.4	29.9	2.1	117.3	....	0.6
3-5	0.180	0.118	6.88	284	118.9	2.86	27.4	35.4	2.3	121.0	....	2.1
3-6	0.195	0.124	7.23	298	118.9	2.89	27.5	39.3	2.4	124.1	....	5.2
3-7	0.246	0.143	7.55	247	127.2	2.69	27.5	47.3	2.6	123.3	3.9	....
3-8	0.257	0.146	7.61	252	126.8	2.65	27.6	48.9	2.6	123.0	3.8	....
3-9	0.279	0.153	8.02	261	126.9	2.68	27.6	54.0	2.7	126.0	0.9	....
8-1	0.114	0.085	5.23	302	102.9	2.73	28.7	20.1	1.7	105.4	....	2.5
8-2	0.128	0.093	5.56	292	106.7	2.74	28.6	23.2	1.9	108.8	....	2.1
8-3	0.139	0.099	5.80	292	108.0	2.74	28.6	25.7	2.0	111.1	....	3.1
8-4	0.152	0.105	6.08	280	112.1	2.75	28.5	28.6	2.1	113.7	....	1.6
8-5	0.164	0.111	6.29	276	113.8	2.74	28.6	31.2	2.2	115.3	....	1.5
8-6	0.175	0.116	6.44	275	114.3	2.72	28.6	33.4	2.2	116.3	....	2.0
8-7	0.184	0.119	6.60	266	117.2	2.71	28.6	35.4	2.3	117.3	....	0.1
8-8	0.193	0.123	6.78	257	120.5	2.72	28.6	37.5	2.3	118.8	1.7	....
8-9	0.204	0.128	6.95	256	121.6	2.71	28.7	39.9	2.4	120.0	1.6	....
8-10	0.220	0.134	6.92	270	115.2	2.60	28.7	41.6	2.5	118.2	....	3.0
8-11	0.233	0.138	6.89	281	110.5	2.52	28.7	42.8	2.5	116.9	....	6.4
8-12	0.242	0.141	6.96	275	111.5	2.49	28.7	44.2	2.6	116.9	....	5.4
8-13	0.243	0.142	7.26	247	122.6	2.60	28.8	46.4	2.6	120.7	1.9	....

TABLE 1.—(*Continued*)

Run[a]	y (ft)	R (ft)	V (ft per sec)	S $\times 10^4$	Observed C	F	t (degrees Centigrade)	R_e $\times 10^{-4}$	c_e	Computed C	Discrepancy −	Discrepancy +
					(*b*) (*Continued*)							
8–14	0.247	0.143	7.37	240	125.8	2.62	28.7	47.4	2.6	121.7	4.1	....
8–15	0.259	0.147	7.33	242	122.8	2.54	28.7	48.5	2.6	120.2	2.6	....
8–16	0.272	0.151	7.64	252	123.8	2.58	28.7	52.0	2.7	122.8	1.0	....
8–17	0.282	0.154	7.80	250	125.6	2.59	28.7	54.1	2.7	123.8	1.8	....
					(*c*) Roughness XI (Fig. 3); Discrepancies by Eq. 17							
45–1	0.083	0.067	0.247	3.6	50.4	0.15	21.3	0.626	1.4	55.8	....	5.4
45–2	0.084	0.067	0.308	4.8	54.1	0.19	21.5	0.793	1.4	55.9	....	1.8
45–3	0.145	0.102	0.420	4.2	64.3	0.19	21.6	1.63	2.0	63.7	0.6	....
45–4	0.200	0.126	0.498	4.0	70.1	0.20	21.5	2.39	2.4	67.8	2.3	....
45–5	0.255	0.146	0.578	4.8	69.1	0.20	21.6	3.22	2.6	70.6	....	1.5
45–6	0.309	0.162	0.644	4.4	76.3	0.20	21.6	3.93	2.7	72.5	3.8	....
45–7	0.379	0.179	0.742	5.2	76.9	0.21	21.8	5.11	2.7	74.2	2.7	....
					(*d*) Roughness I (Fig. 3); Discrepancies by Eq. 17							
44–1	0.098	0.076	0.218	2.6	49.0	0.12	22.5	0.648	1.5	46.7	2.3	....
44–2	0.142	0.100	0.311	3.6	51.7	0.15	22.5	1.22	2.0	51.9	....	0.2
44–3	0.182	0.119	0.412	4.4	57.1	0.17	22.5	1.91	2.3	55.2	1.9	....
44–4	0.254	0.146	0.460	4.6	56.2	0.16	22.5	2.61	2.6	59.0	....	2.8
44–5	0.324	0.166	0.602	5.4	63.6	0.19	21.3	3.79	2.7	61.4	2.2	....
44–6	0.381	0.180	0.583	4.6	64.1	0.17	21.4	3.99	2.7	62.8	1.3	....
25–1	0.150	0.104	0.752	19.8	52.4	0.34	24.2	3.19	2.0	52.6	....	0.2
25–2	0.231	0.138	0.994	20.4	59.3	0.36	24.2	5.56	2.5	57.9	1.4	....
25–3	0.270	0.151	1.06	19.6	61.7	0.36	24.2	6.49	2.6	59.6	2.1	....
25–4	0.345	0.171	1.25	21.4	65.1	0.37	24.3	8.71	2.7	62.0	3.1	....
22–1	0.152	0.105	1.46	78	50.8	0.66	26.7	6.58	2.1	52.9	....	2.1
22–2	0.193	0.123	1.80	79	57.7	0.72	26.7	9.55	2.3	55.8	1.9	....
22–3	0.257	0.146	2.16	79	63.6	0.75	26.7	13.6	2.6	59.1	4.5	....
22–4	0.341	0.170	2.31	79	63.0	0.70	26.7	16.9	2.7	61.8	1.2	....
4–1	0.167	0.112	3.27	306	55.9	1.41	28.1	16.3	2.2	54.1	1.8	....
4–2	0.219	0.133	3.57	309	55.7	1.34	28.4	21.3	2.5	57.4	....	1.7
4–3	0.262	0.148	3.81	312	56.0	1.31	28.3	25.1	2.6	59.3	....	3.3
4–4	0.300	0.159	4.00	312	56.8	1.29	28.3	28.5	2.7	60.7	....	3.9
					(*e*) Roughness II (Fig. 3); Discrepancies by Eq. 17							
43–1	0.093	0.073	0.198	4.5	34.5	0.11	22.3	0.564	1.5	35.2	....	0.7
43–2	0.130	0.094	0.243	4.0	39.6	0.12	22.3	0.890	1.9	40.0	....	0.4
43–3	0.199	0.126	0.368	5.0	46.5	0.15	22.3	1.80	2.4	45.6	0.9	....
43–4	0.240	0.141	0.345	3.6	48.5	0.12	22.3	1.88	2.6	47.7	0.8	....
43–5	0.317	0.164	0.509	5.5	53.6	0.16	22.4	3.25	2.6	50.4	3.2	....
43–6	0.391	0.182	0.556	5.8	54.0	0.16	22.4	3.94	2.6	52.2	1.8	....
26–5	0.092	0.072	0.410	20.4	33.6	0.24	23.0	1.17	1.5	35.1	....	1.5
26–6	0.134	0.096	0.612	21.6	42.5	0.29	23.0	2.32	1.9	40.4	2.1	....
26–1	0.168	0.112	0.718	21.2	46.3	0.31	22.2	3.13	2.2	43.4	2.9	....
26–2	0.238	0.140	0.874	21.8	49.9	0.32	22.9	4.83	2.5	47.6	2.3	....
26–3	0.277	0.153	0.911	20.8	51.2	0.30	23.0	5.50	2.7	49.3	1.9	....
26–4	0.339	0.170	1.03	21.6	53.4	0.31	23.1	6.90	2.7	51.1	2.3	....
21–5	0.108	0.082	0.959	78	37.9	0.52	26.5	3.37	1.7	37.5	0.4	....
21–6	0.134	0.096	1.11	79	40.3	0.54	26.6	4.60	1.9	40.4	....	0.1
21–1	0.150	0.104	1.18	77	41.7	0.54	26.3	5.25	2.0	41.9	....	0.2
21–7	0.178	0.117	1.33	78	44.0	0.56	26.7	6.69	2.2	44.1	....	0.1
21–2	0.190	0.122	1.35	82	42.7	0.55	26.4	7.04	2.3	45.0	....	2.3
21–8	0.247	0.143	1.56	79	46.5	0.55	26.7	9.62	2.6	48.0	....	1.5
21–3	0.255	0.146	1.60	79	47.2	0.56	26.4	9.96	2.6	48.4	....	1.2
21–9	0.313	0.163	1.75	78	49.1	0.55	26.8	12.3	2.7	50.4	....	1.3
21–4	0.319	0.165	1.93	78	54.0	0.60	26.5	13.6	2.7	50.6	3.4	....

TABLE 1.—(*Continued*)

Run[a]	y (ft)	R (ft)	V (ft per sec)	S ×10⁴	Observed C	F	t (degrees Centigrade)	R_e ×10⁻⁴	c_e	Computed C	Discrepancy −	Discrepancy +
					(*e*) (*Continued*)							
5–1	0.171	0.114	2.71	310	45.6	1.06	28.4	13.8	2.2	43.6	2.0	....
5–2	0.236	0.139	3.02	310	46.0	1.10	28.6	18.9	2.5	47.5	....	1.5
5–3	0.287	0.156	3.24	312	46.3	1.06	28.7	22.7	2.7	49.6	....	3.3
5–4	0.331	0.168	3.41	310	47.3	1.04	28.6	25.7	2.7	50.9	....	3.6
					(*f*) Roughness III (Fig. 3); Discrepancies by Eq. 17							
42–1	0.084	0.067	0.160	5.1	27.3	0.10	21.3	0.410	1.4	26.7	0.6	....
42–2	0.139	0.099	0.229	4.9	32.9	0.11	21.6	0.864	2.0	34.0	....	1.1
42–3	0.191	0.122	0.297	4.8	38.7	0.12	22.0	1.40	2.3	38.0	0.7	....
42–4	0.253	0.145	0.389	6.8	39.2	0.14	22.0	2.18	2.6	41.3	....	2.1
42–5	0.328	0.167	0.414	5.6	42.8	0.13	22.2	2.68	2.7	43.8	....	1.0
42–6	0.378	0.179	0.392	4.5	43.7	0.11	22.2	2.72	2.7	45.0	....	1.3
27–1	0.105	0.080	0.447	21.2	34.3	0.24	22.9	1.41	1.6	30.0	4.3	....
27–2	0.154	0.106	0.578	22.7	37.3	0.26	23.0	2.42	2.1	35.3	2.0	....
27–3	0.185	0.120	0.662	22.0	40.8	0.27	23.0	3.14	2.3	37.6	3.2	....
27–4	0.259	0.147	0.800	23.2	43.3	0.28	23.1	4.66	2.6	41.5	1.8	....
27–5	0.307	0.161	0.880	25.0	43.8	0.28	23.0	5.61	2.7	43.2	0.6	....
20–1	0.141	0.100	1.11	77	40.1	0.52	26.1	4.71	2.0	34.1	6.0	....
20–2	0.186	0.120	1.27	78	41.5	0.52	26.1	6.50	2.3	37.7	3.8	....
20–3	0.233	0.138	1.42	77	43.7	0.52	26.2	8.39	2.5	40.3	3.4	....
20–4	0.306	0.161	1.59	79	44.6	0.51	26.2	10.9	2.7	43.2	1.4	....
6–1	0.142	0.100	2.01	309	36.3	0.94	28.5	9.06	2.0	34.2	2.1	....
6–2	0.189	0.122	2.43	309	39.6	0.98	28.5	13.2	2.3	37.9	1.7	....
6–3	0.232	0.138	2.74	309	41.9	1.00	28.5	16.9	2.5	40.3	1.6	....
6–4	0.284	0.155	3.00	309	43.4	0.99	28.6	20.9	2.7	42.5	0.9	....
6–5	0.316	0.164	3.18	309	44.7	1.00	28.6	23.4	2.7	43.5	1.2	....
					(*g*) Roughness IV (Fig. 3); Discrepancies by Eq. 17							
41–1	0.087	0.069	0.139	4.4	25.2	0.08	22.1	0.372	1.4	26.6	....	1.4
41–2	0.146	0.102	0.255	6.0	32.6	0.12	22.1	1.01	2.0	34.0	....	1.4
41–3	0.196	0.124	0.263	4.3	36.0	0.10	22.0	1.26	2.4	37.8	....	1.8
41–4	0.259	0.147	0.360	5.8	39.0	0.13	22.0	2.04	2.6	40.9	....	1.9
41–5	0.320	0.165	0.333	5.4	35.3	0.10	22.0	2.12	2.7	43.0	....	7.7
41–6	0.381	0.180	0.387	5.2	40.1	0.11	22.1	2.69	2.7	44.5	....	4.4
28–1	0.099	0.077	0.346	21.0	27.3	0.19	21.6	1.01	1.6	28.6	....	1.3
28–2	0.143	0.101	0.496	17.5	37.4	0.24	21.9	1.93	2.0	33.7	3.7	....
28–3	0.197	0.125	0.623	20.4	39.0	0.25	22.0	3.00	2.4	37.8	1.2	....
28–4	0.274	0.152	0.756	21.8	41.6	0.26	22.2	4.45	2.7	41.6	0.0	....
28–5	0.348	0.172	0.816	21.6	42.3	0.24	22.2	5.44	2.7	43.7	....	1.4
19–1	0.131	0.095	1.03	75	38.6	0.50	26.1	4.13	1.9	32.5	6.1	....
19–2	0.189	0.122	1.26	75	41.6	0.51	26.2	6.50	2.3	37.3	4.3	....
19–3	0.242	0.141	1.41	75	43.4	0.51	26.2	8.50	2.6	40.2	3.2	....
19–4	0.342	0.170	1.61	76	44.7	0.48	26.2	11.7	2.7	43.6	1.1	....
7–1	0.203	0.127	2.42	309	38.6	0.95	28.6	13.8	2.4	38.2	0.4	....
7–2	0.235	0.139	2.64	310	40.2	0.96	28.7	16.5	2.5	39.8	0.4	....
7–3	0.263	0.148	2.82	312	41.4	0.96	28.7	18.8	2.6	41.0	0.4	....
7–4	0.290	0.157	2.93	310	42.1	0.96	28.8	20.7	2.7	42.1	0.0	....
7–5	0.316	0.164	3.04	312	42.5	0.95	28.8	22.4	2.7	42.9	....	0.4
					(*h*) Roughness V (Fig. 3); Discrepancies by Eq. 17							
40–1	0.094	0.074	0.179	4.8	30.1	0.10	22.4	0.514	1.5	30.5	....	0.4
40–2	0.146	0.102	0.238	4.4	35.5	0.11	22.3	0.944	2.0	36.7	....	1.2
40–3	0.226	0.136	0.382	5.8	43.0	0.14	22.2	1.27	2.5	42.1	0.9	....
40–4	0.190	0.122	0.297	4.3	41.0	0.12	22.2	1.40	2.3	40.1	0.9	....
40–5	0.274	0.152	0.384	5.1	43.6	0.13	22.2	2.26	2.7	44.3	....	0.7
40–6	0.319	0.165	0.379	4.4	44.5	0.12	22.2	2.42	2.7	45.7	....	1.2
40–7	0.393	0.182	0.447	5.2	45.9	0.13	22.2	3.16	2.7	47.4	....	1.5

TABLE 1.—(*Continued*)

Run[a]	y (ft)	R (ft)	V (ft per sec)	S $\times 10^4$	Observed C	F	t (degrees Centigrade)	R_e $\times 10^{-4}$	c_s	Computed C	Discrepancy −	Discrepancy +
						(*h*) (*Continued*)						
29–1	0.110	0.083	0.379	19.0	30.2	0.20	22.3	1.22	1.7	32.8	....	2.6
29–2	0.148	0.103	0.540	19.6	38.0	0.25	22.4	2.17	2.0	36.8	1.2	....
29–3	0.186	0.120	0.642	20.2	41.2	0.26	22.4	3.01	2.3	39.8	1.4	....
29–4	0.287	0.156	0.765	18.8	44.7	0.25	22.6	4.66	2.7	44.7	0.0	....
29–5	0.330	0.168	0.869	21.0	46.3	0.27	22.6	5.70	2.7	46.0	0.3	....
18–1	0.171	0.114	0.981	71	34.5	0.42	25.8	4.71	2.2	38.7	....	4.2
18–3	0.144	0.101	1.12	78	39.8	0.52	25.9	4.78	2.0	36.5	3.3	....
18–2	0.204	0.128	1.37	79	43.3	0.54	25.8	7.40	2.4	40.9	2.4	....
18–4	0.256	0.146	1.54	78	45.6	0.53	25.9	9.50	2.6	43.5	2.1	....
18–5	0.332	0.168	1.70	79	46.6	0.52	26.0	12.1	2.7	46.0	0.6	....
9–1	0.199	0.126	2.56	309	41.1	1.01	28.5	14.4	2.4	40.7	0.4	....
9–2	0.246	0.143	2.86	309	43.0	1.01	28.4	18.2	2.6	43.1	....	0.1
9–3	0.346	0.172	3.29	312	45.0	0.99	28.5	25.3	2.7	46.4	....	1.4
						(*i*) Roughness VI (Fig. 3); Discrepancies by Eq. 17						
39–1	0.102	0.078	0.197	4.6	32.8	0.10	22.1	0.600	1.6	35.2	....	2.4
39–2	0.127	0.092	0.238	4.4	37.3	0.11	22.1	0.851	1.8	38.2	....	0.9
39–3	0.211	0.130	0.369	5.2	44.8	0.14	22.0	1.86	2.4	44.8	0.0	....
39–4	0.247	0.143	0.380	5.4	43.2	0.14	22.1	2.10	2.6	46.6	....	3.4
39–5	0.321	0.165	0.494	5.8	50.5	0.15	22.1	3.16	2.7	49.2	1.3	....
39–6	0.405	0.185	0.573	6.9	50.7	0.16	22.1	4.10	2.7	51.2	....	0.5
30–1	0.106	0.081	0.396	19.2	31.8	0.21	22.1	1.24	1.6	35.7	....	3.9
30–2	0.152	0.105	0.604	20.8	40.8	0.27	22.2	2.46	2.1	40.8	0.0	....
30–3	0.205	0.128	0.734	20.8	45.0	0.29	22.3	3.65	2.4	44.5	0.5	....
30–4	0.285	0.155	0.838	20.0	47.6	0.28	22.4	5.06	2.7	48.1	....	0.5
30–5	0.329	0.167	0.919	21.0	49.0	0.28	22.3	5.97	2.7	49.4	....	0.4
17–1	0.155	0.106	1.24	85	41.4	0.56	26.0	5.62	2.1	41.0	0.4	....
17–2	0.195	0.124	1.52	80	48.3	0.61	26.0	7.99	2.3	43.8	4.5	....
17–3	0.245	0.142	1.71	84	49.4	0.61	25.9	10.5	2.6	46.5	2.9	....
17–4	0.316	0.164	1.97	83	53.3	0.62	25.9	13.6	2.7	49.1	4.2	....
10–1	0.187	0.121	2.63	312	42.8	1.07	27.8	14.0	2.3	43.4	....	0.6
10–2	0.250	0.144	3.08	311	46.0	1.09	27.8	19.6	2.6	46.7	....	0.7
10–3	0.325	0.166	3.44	310	48.0	1.06	28.0	25.4	2.7	49.3	....	1.3
						(*j*) Roughness VII (Fig. 3); Discrepancies by Eq. 17						
38–1	0.107	0.081	0.271	4.4	45.3	0.15	21.7	0.847	1.6	42.2	3.1	....
38–2	0.155	0.106	0.300	4.6	42.9	0.13	21.9	1.23	2.1	47.4	....	4.5
38–3	0.192	0.123	0.339	4.4	46.1	0.13	21.9	1.60	2.3	50.1	....	4.0
38–4	0.242	0.141	0.422	5.4	48.3	0.14	21.9	2.30	2.6	52.8	....	4.5
38–5	0.312	0.163	0.443	4.6	51.2	0.14	21.9	2.78	2.7	55.4	....	4.2
38–6	0.386	0.181	0.558	6.2	52.7	0.16	21.9	3.88	2.7	57.2	....	4.5
31–1	0.098	0.076	0.412	20.2	33.2	0.23	22.6	1.29	1.6	41.1	....	7.9
31–2	0.133	0.096	0.595	20.2	42.8	0.29	22.6	2.23	1.9	45.3	....	2.5
31–3	0.204	0.128	0.818	20.9	50.1	0.32	22.5	4.08	2.4	50.8	....	0.7
31–4	0.268	0.150	0.909	21.4	50.8	0.31	22.6	5.34	2.6	53.8	....	3.0
31–5	0.329	0.167	0.996	21.4	52.6	0.31	22.5	6.51	2.7	55.8	....	3.2
16–1	0.103	0.079	1.20	80	47.5	0.66	25.9	4.00	1.6	41.7	5.8	....
16–2	0.147	0.103	1.47	78	52.0	0.68	25.9	6.39	2.0	46.7	5.3	....
16–3	0.196	0.124	1.72	81	54.1	0.68	26.0	9.05	2.4	50.4	3.7	....
16–4	0.232	0.138	1.90	82	56.4	0.69	26.0	11.1	2.5	52.3	4.1	....
16–5	0.306	0.161	2.09	83	57.2	0.67	26.0	14.3	2.7	55.2	2.0	....
11–1	0.119	0.088	2.12	312	40.5	1.09	27.6	8.23	1.8	43.8	....	3.3
11–2	0.154	0.106	2.56	311	44.6	1.15	27.6	11.9	2.1	47.3	....	2.7
11–3	0.206	0.128	3.08	310	48.9	1.20	27.6	17.4	2.4	50.9	....	2.0
11–4	0.266	0.149	3.49	312	51.1	1.19	27.7	22.9	2.6	53.8	....	2.7
11–5	0.325	0.166	3.78	309	52.7	1.17	27.7	27.6	2.7	55.7	....	3.0

TABLE 1.—(*Continued*)

Runs	y (ft)	R (ft)	V (ft per sec)	S $\times 10^4$	Observed C	F	t (degrees Centigrade)	R_e $\times 10^{-4}$	c_s	Computed C	Discrepancy −	Discrepancy +
(*k*) Roughness VIII (Fig. 3); Discrepancies by Eq. 17												
37-1	0.083	0.067	0.254	4.6	45.8	0.16	21.2	0.640	1.4	45.6	0.2	....
37-2	0.130	0.094	0.278	3.4	49.2	0.14	21.2	0.992	1.9	52.0	....	2.8
37-3	0.181	0.118	0.422	4.8	56.0	0.18	21.3	1.89	2.3	56.4	....	0.4
37-4	0.257	0.146	0.543	5.6	59.9	0.19	21.3	3.02	2.6	60.4	....	0.5
37-5	0.322	0.165	0.628	6.8	59.2	0.20	21.4	3.95	2.7	62.6	....	3.4
37-6	0.388	0.181	0.724	7.6	61.7	0.20	21.4	4.99	2.7	64.2	....	2.5
32-1	0.097	0.076	0.561	20.8	44.8	0.32	21.1	1.60	1.5	47.8	....	3.0
32-2	0.133	0.096	0.706	20.0	51.0	0.34	21.1	2.55	1.9	52.3	....	1.3
32-3	0.202	0.127	0.926	20.2	57.9	0.36	21.3	4.46	2.4	57.7	0.2	....
32-4	0.245	0.142	0.962	20.5	56.3	0.34	21.8	5.26	2.6	59.9	....	3.6
32-5	0.320	0.165	1.14	20.1	62.7	0.36	21.9	7.25	2.7	62.6	0.1	....
15-1	0.372	0.178	2.41	78	64.8	0.70	25.7	18.1	2.7	63.9	0.9	....
12-1	0.111	0.084	2.60	312	50.8	1.37	27.0	9.43	1.7	49.8	1.0	....
12-2	0.151	0.105	3.20	311	56.1	1.45	27.1	14.5	2.1	54.1	2.0	....
12-3	0.195	0.124	3.72	310	60.0	1.49	27.1	20.0	2.3	57.2	2.8	....
12-4	0.246	0.143	4.12	312	61.7	1.46	27.2	25.6	2.6	60.0	1.7	....
12-5	0.321	0.165	4.55	311	63.4	1.41	27.2	32.7	2.7	62.6	0.8	....
(*l*) Roughness IX (Fig. 3); Discrepancies by Eq. 17												
36-4	0.096	0.075	0.325	4.2	57.9	0.19	21.1	0.92	1.5	58.5	....	0.6
36-5	0.155	0.106	0.466	4.5	67.3	0.21	21.1	1.87	2.1	65.2	2.1	....
36-6	0.200	0.126	0.504	4.3	68.5	0.20	21.1	2.40	2.4	68.4	0.1	....
36-1	0.245	0.142	0.547	4.6	67.6	0.20	21.0	2.93	2.6	70.7	....	3.1
36-7	0.248	0.144	0.571	4.0	75.3	0.20	21.1	3.09	2.6	70.9	4.4	....
36-2	0.334	0.169	0.678	4.4	78.7	0.20	21.1	4.32	2.7	73.8	4.9	....
36-3	0.404	0.185	0.725	5.0	75.4	0.19	21.0	5.04	2.7	75.4	0.0	....
33-1	0.103	0.079	0.682	20.8	53.1	0.38	21.4	2.05	1.6	59.3	....	6.2
33-2	0.132	0.095	0.809	20.6	57.8	0.39	21.7	2.95	1.9	63.0	....	5.2
33-3	0.153	0.106	0.906	19.2	63.6	0.41	21.5	3.65	2.1	65.0	....	1.4
33-4	0.209	0.130	1.13	21.2	68.4	0.44	21.8	5.64	2.4	68.9	....	0.5
33-5	0.263	0.148	1.26	20.4	72.4	0.43	21.8	7.18	2.6	71.4	1.0	....
33-6	0.348	0.172	1.40	19.6	76.4	0.42	21.9	9.28	2.7	74.1	2.3	....
13-1	0.126	0.092	3.51	312	65.5	1.74	27.0	14.0	1.8	62.4	3.1	....
13-2	0.158	0.108	3.84	312	66.1	1.70	27.0	17.9	2.1	65.4	0.7	....
13-3	0.202	0.127	4.20	312	66.7	1.66	27.1	23.1	2.4	68.5	....	1.8
13-4	0.268	0.150	4.60	312	67.2	1.56	27.1	29.9	2.6	71.6	....	4.4
13-5	0.324	0.166	4.90	310	68.3	1.52	26.9	35.2	2.7	73.5	....	5.2
(*m*) Roughness X (Fig. 3); Discrepancies by Eq. 17												
35-1	0.103	0.079	0.422	4.0	75.1	0.23	21.2	1.26	1.6	70.9	4.2	....
35-2	0.139	0.099	0.486	4.4	73.7	0.23	20.8	1.80	2.0	75.2	....	1.5
35-3	0.203	0.127	0.676	5.4	81.5	0.27	20.8	3.22	2.4	80.0	1.5	....
35-4	0.252	0.145	0.745	5.8	81.2	0.26	20.8	4.04	2.6	82.4	....	1.2
35-5	0.320	0.165	0.855	5.8	87.4	0.27	20.9	5.30	2.7	84.8	2.6	....
35-6	0.394	0.182	0.882	5.8	85.7	0.25	20.9	6.05	2.7	86.6	....	0.9
34-1	0.106	0.081	0.746	18.0	61.9	0.40	21.7	2.31	1.6	71.3	....	9.4
34-2	0.161	0.109	1.12	20.4	75.2	0.49	21.8	4.71	2.1	77.0	....	1.8
34-3	0.210	0.130	1.30	19.4	82.0	0.50	21.9	6.51	2.4	80.3	1.7	....
34-4	0.261	0.148	1.44	19.2	84.8	0.50	21.9	8.16	2.6	82.8	2.0	....
34-5	0.326	0.166	1.52	18.3	87.3	0.47	21.9	9.78	2.7	85.0	2.3	....
14-1	0.120	0.089	3.83	305	73.7	1.95	27.0	14.73	1.8	73.1	0.6	....
14-2	0.142	0.100	4.49	309	80.7	2.10	27.0	19.50	2.0	75.4	5.3	....
14-3	0.194	0.124	4.91	310	79.3	1.96	27.0	26.3	2.3	79.4	....	0.1
14-4	0.239	0.140	5.19	308	78.9	1.87	27.0	31.6	2.6	81.9	....	3.0
14-5	0.300	0.159	5.60	304	80.4	1.80	27.1	38.7	2.7	84.2	....	3.8

TABLE 1.—(*Continued*)

Run[a]	y (ft)	R (ft)	V (ft per sec)	S $\times 10^4$	Observed C	F	t (degrees Centigrade)	Re $\times 10^{-4}$	c_a	Computed C	Discrepancy −	Discrepancy +
(*n*) Bazin's Tests on Plank Flume with Battens (Flume Widths—Setups 12, 13, and 15, 6.43 Ft; Setups 14 and 17, 6.40 Ft; and Setup 16, 6.44 Ft)[a]												
12–1	0.33	0.302	1.65	15	77.4	0.50	8.5	13.5	0.6	77.4	0.0	....
12–2	0.51	0.442	2.17	15	84.5	0.54	8.5	26.6	1.0	84.4	0.1	....
12–3	0.89	0.634	2.86	15	91.0	0.54	8.5	49.3	1.4	91.1	....	0.1
12–4	1.02	0.775	3.33	15	94.0	0.58	8.5	70.1	1.7	94.9	....	0.9
12–5	1.23	0.889	3.68	15	97.0	0.59	8.5	88.9	1.9	97.5	....	0.5
12–6	1.42	0.986	3.98	15	99.0	0.59	8.5	107	2.1	99.5	....	0.5
12–7	1.62	1.706	4.19	15	99.0	0.58	8.5	122	2.2	101.1	....	2.1
13–1	0.22	0.205	2.50	59	71.8	0.94	7.0	13.3	0.4	70.5	1.3	....
13–2	0.33	0.302	3.34	59	79.0	1.03	7.0	26.2	0.6	77.4	1.6	....
13–3	0.51	0.442	4.40	59	86.2	1.09	7.0	50.6	1.0	84.4	1.8	....
13–4	0.67	0.552	5.08	59	89.0	1.10	7.0	72.9	1.2	88.5	0.5	....
13–5	0.80	0.643	5.63	59	91.4	1.11	7.0	94.1	1.4	91.3	0.1	....
13–6	0.92	0.716	6.14	59	94.5	1.13	7.0	114	1.5	93.3	1.2	....
13–7	1.05	0.790	6.48	59	94.8	1.12	7.0	133	1.7	95.2	....	0.4
14–1	0.19	0.182	2.85	89	70.8	1.15	14.0	16.4	0.3	68.3	2.5	....
14–2	0.30	0.273	3.75	89	76.4	1.21	14.0	32.4	0.6	75.6	0.8	....
14–3	0.46	0.403	4.92	89	82.4	1.28	14.0	62.7	0.9	82.7	....	0.3
14–4	0.59	0.499	5.77	89	86.8	1.32	14.0	91.0	1.1	86.6	0.2	....
14–5	0.71	0.582	6.38	89	88.9	1.33	14.0	117	1.3	89.5	....	0.6
14–6	0.83	0.658	6.86	89	89.9	1.34	14.0	143	1.4	91.7	....	1.8
14–7	0.94	0.726	7.26	89	90.5	1.32	14.0	167	1.6	93.6	....	3.1
15–1	0.43	0.378	1.28	15	53.7	0.34	16.5	16.3	0.8	50.5	3.2	....
15–2	0.66	0.550	1.68	15	58.6	0.36	16.5	31.1	1.2	57.4	1.2	....
15–3	1.02	0.777	2.21	15	64.8	0.39	16.5	57.9	1.7	63.9	0.9	....
15–4	1.33	0.942	2.55	15	67.8	0.39	16.5	80.9	2.0	67.6	0.2	....
15–5	1.61	1.073	2.81	15	70.1	0.39	16.5	102	2.2	70.0	0.1	....
15–6	1.91	1.197	2.97	15	70.0	0.38	16.5	120	2.4	72.2	....	2.2
15–7	2.18	1.299	3.11	15	70.5	0.37	16.5	136	2.5	73.7	....	3.2
16–1	0.29	0.264	1.91	59	48.3	0.62	7.5	13.3	0.6	44.1	4.2	....
16–2	0.44	0.384	2.56	59	53.7	0.68	7.5	25.9	0.8	50.8	2.9	....
16–3	0.67	0.553	3.37	59	59.0	0.73	7.5	49.2	1.2	57.5	1.5	....
16–4	0.89	0.686	3.88	59	61.0	0.73	7.5	70.3	1.5	61.6	....	0.6
16–5	1.05	0.791	4.31	59	63.1	0.74	7.5	90.0	1.7	64.3	....	1.2
16–6	1.21	0.882	4.65	59	64.5	0.75	7.5	108	1.9	66.3	....	1.8
16–7	1.38	0.965	4.91	59	65.1	0.74	7.5	125	2.0	68.0	....	2.9
17–1	0.25	0.232	2.21	88.6	48.7	0.78	16.5	17.3	0.5	41.7	7.0	....
17–2	0.39	0.350	2.85	88.6	51.2	0.81	16.5	33.6	0.8	49.2	2.0	....
17–3	0.60	0.509	3.75	88.6	55.8	0.85	16.5	64.3	1.1	56.0	....	0.2
17–4	0.78	0.628	4.37	88.6	58.6	0.88	16.5	92.5	1.3	59.8	....	1.2
17–5	0.94	0.725	4.85	88.6	60.5	0.88	16.5	118	1.6	62.6	....	2.1
17–6	1.09	0.812	5.22	88.6	61.5	0.88	16.5	143	1.7	64.7	....	3.2
17–7	1.22	0.885	5.57	88.6	62.9	0.89	16.5	166	1.9	66.4	....	3.5

[a] The notation 46–1, for example, refers to run 1 in setup 46. In Table 1 (*n*), setups 12 to 17, the run numbers are as actually reported by Bazin.

TABLE 2.—Numerical Values of Roughnesses

Roughness (see Fig. 3)	ϵ (ft)	Actual size h (ft)	$\frac{\epsilon}{h}$	Spacing s (ft)	$\frac{s}{h}$
(*a*) Writer's Observations					
XI	0.00292	0.02083	0.140	3.333	160
I	0.00564	0.02083	0.271	1.667	80
II	0.01048	0.02083	0.503	0.833	40
III	0.01568	0.02083	0.753	0.417	20
IV	0.01603	....	....	0.208	....
V	0.01372	....	....	0.208	....
VI	0.01128	....	....	0.208	....
VII	0.00784	0.01042	0.753	0.208	20
VIII	0.00524	0.01042	0.503	0.417	40
IX	0.00282	0.01042	0.271	0.833	80
X	0.00146	0.01042	0.140	1.677	160
(*b*) Bazin's Data (Table 1(*n*))					
Setups 12, 13, 14	0.00363	0.0328	0.111	0.121[a]	3.70
Setups 15, 16, 17	0.0216	0.0328	0.659	0.253[b]	7.70

[a] Battens spaced closely. [b] Battens placed wide apart

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

CONVERSION OF KINETIC TO POTENTIAL ENERGY IN FLOW EXPANSIONS

By A. A. Kalinske,[1] Assoc. M. Am. Soc. C. E.

Synopsis

Information on the energy changes that occur when fluids flow in expanding conduits is presented in this paper. Specific data are presented on mean velocity distribution, turbulence, and pressure changes for 7.5°, 15°, 30°, and 180° expansions from 2.75-in. to 4.75-in. pipes. The velocity and turbulence measurements were made by photographing, on motion-picture film, the flow in transparent conduits. The water therein had suspended in it small, immiscible droplets which formed streaks on the pictures, from which the direction and magnitude of the velocity at any instant in two dimensions could be obtained.

The analysis of the data for mean velocity distribution, variation of turbulence, and pressure changes gives considerable information on the mechanics of the energy conversion and dissipation processes. It is found that the total loss of energy is considerably greater than the total energy of turbulence that is produced, thus indicating that a major portion of the energy is lost by direct conversion into heat at the regions of high, local shear in the fluid. The turbulence created is a by-product of the energy dissipation process; however, it plays an important rôle in the shaping of the mean velocity distribution curve in and beyond the expansion. Data are presented on the energy loss and efficiency of the energy conversion for two rates of flow through the four expansions studied.

A. Introduction

Examples of fluid flow in conduits where the cross-sectional area increases, with a resultant decrease in velocity and increase in pressure, are common wherever fluids are carried and controlled. Specific examples range all the

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by May 1, 1945.

[1] Associate Prof., Univ. of Iowa, and Associate Director, Iowa Inst. of Hydr. Research, Iowa City, Iowa.

way from small pipe fittings, known as "reducers," to draft tubes on turbines, and the loss of head occurring in any such expansion may be only of minor importance to the engineer; or it may be of considerable importance. Throughout the years, considerable data have been accumulated, relating to losses in various types of such expansions, and in general hydraulic engineers have a fair idea of the pressure losses involved. Therefore, this paper is not concerned with the mere presentation of further pressure loss data, but rather with supplying data that will shed some light on the internal mechanics of the energy changes that occur in the fluid flowing in and beyond expansions.

The experimental data presented herein have been an outgrowth of a study relating to the general problem of the hydraulic conversion of kinetic to potential energy, which was started by the Society's Committee on Hydraulic Research in 1935 at the Iowa Institute of Hydraulic Research. In order that quantitative information might be available relating to the conversion process, it was necessary that all the elements entering into the process be measured. Therefore, for each expansion studied the mean velocity distribution was determined at several sections in and beyond the expansion. Since the conversion process involves transformation of a part of the kinetic energy into turbulence energy, it was desirable to have some measurements relating to the intensity of turbulence at various sections. This, of course, proved to be the most difficult part of the experimental study; however, some very significant data were secured. The change in the pressure along the conduit in which the expansion was installed was also determined.

Before the experiments and results are discussed in detail, a brief review will be given of some of the fundamental facts known about flow in expansions and pertinent past work on this subject. Also, since certain items relating to fluid turbulence are basic to this particular study, a short description will be given of the relation of turbulence to the energy changes that occur in flowing fluids.

Notation

The letter symbols, used in this paper, conform essentially to American Standard Letter Symbols for Hydraulics,[2] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1942.

B. Mechanics of Fluid Flow in Expansions

From a practical standpoint the principal item of interest is the pressure loss that occurs when fluids flow through expansions. However, in order to understand exactly what causes this extra pressure loss and where it takes place along the conduit, it is necessary that the flow mechanism be examined in detail. This paper is concerned only with incompressible fluids; therefore, density and temperature changes will be neglected.

Visual observations of flow in conical expansions reveal that there is an appreciable increase in the angularity of the velocity of fluid particles near the boundaries as they pass through an expansion. In other words, the flow

[2] ASA—Z10.2—1942.

becomes more eddying, and the eddies created near the boundaries gradually "eat" into the center stream which for a while after leaving the smaller conduit is relatively free of turbulence. The eddies coming from the wall naturally have relatively low forward velocities, and, as they intermingle with the rapidly moving central part of the stream, the fast-moving fluid is gradually slowed down. As the mean velocity decreases there is a gradual pressure rise, and the efficiency of the conversion process of kinetic to potential energy depends on how much of the initial kinetic energy is used in forming eddies and in direct frictional loss.

The loss in any expanding section can be divided into two parts—that due to friction on the boundaries and the extra loss due to the shape and form of the conduit. Except for slow expansions (those having total angles less than about 10°) the frictional loss is negligible compared to what might be called "form loss." Both analytical work and experimental data indicate that the total expansion angle giving the least loss varies between 5° and 10°, the exact value being dependent on the velocity ratio and the initial velocity distribution. As the expansion angle is increased, the form loss increases rapidly, as it is associated with the flow separation that occurs at the boundaries; that is, as the expansion angle becomes large, the fluid near the boundaries does not have sufficient kinetic energy to move against the pressure gradient; instead a backward flow occurs. There is thus created a discontinuity inside the fluid with the result that high shearing stresses occur in the fluid and violent turbulence is generated.

Some extensive data on the loss in expansions varying from 3° to 180° (sudden expansion) between circular pipes of 1.5 and 3.0 in. in diameter were obtained by A. H. Gibson.[3] He expressed the loss in terms of the Borda loss for sudden expansions, and showed that the total loss was a minimum for an expansion of 5.5°, being about 13.5% of the Borda loss; it reached a maximum of 121% when the angle was 63°, and then diminished to 102% for a 180° expansion. Extensive experiments on the loss in sudden expansions were reported by W. H. Archer,[4] who expressed the loss as

$$h_i = \frac{1.098\,(\overline{U}_{m1} - \overline{U}_{m2})^{1.919}}{2\,g} \qquad (1)$$

in which $\overline{U}_{m1}$ and $\overline{U}_{m2}$ are the mean velocities in the upstream and downstream conduits, respectively.

In his work on conversion of energy in cross-sectional divergences, H. Peters[5] emphasized the influence of the conditions of inflow on the efficiency of the conversion process. He stated that complete conversion requires a discharge length beyond the expansion which depends on the angle and the velocity distribution at the entrance to the expansion. He defined the efficiency of conversion as the ratio of the actual rise in pressure, corrected for wall

[3] "Conversion of Kinetic to Potential Energy in the Flow of Water Through Passages Having Divergent Boundaries," by A. H. Gibson, *Engineering*, Vol. 93, 1912, p. 205.

[4] "Experimental Determination of Loss of Head Due to Sudden Enlargement in Circular Pipes," by W. H. Archer, *Transactions*, Am. Soc. C. E., Vol. LXXVI (1913), p. 999.

[5] "Conversion of Energy in Cross-Sectional Divergences Under Different Conditions of Inflow," by H. Peters, *Technical Memorandum No. 737* (Translation), National Advisory Committee for Aeronautics, 1934.

friction, to the difference in the initial and final kinetic energies. The kinetic energy was calculated by integration of the actual velocity distribution across any section. Mr. Peters showed that superimposing a spiral motion on the flow entering any expansion increased the efficiency markedly. It appears that the rotational motion delays separation, and thus decreases the energy loss.

A. P. Kratz and J. R. Fellows[6] conducted detailed experiments on pressure losses resulting from changes in the cross-sectional area of rectangular air ducts, and among their findings is the conclusion that diverging sections having included angles greater than 60° give losses at least as great as those in a sudden expansion, and that no material advantage is gained by use of angles greater than 30°.

J. Nikuradse[7] made velocity distribution measurements in two-dimensional diverging sections up to angles of 5°, and it is apparent from his data that the shape of the velocity curves depends on the angle of divergence, the curves becoming more pointed for the larger angles. He noted that separation occurred at a 5° divergence and that, for slightly larger angles, there was a tendency for asymmetrical velocity distribution and a switching of the flow from wall to wall.

Some analytical work has been done in regard to retarded and accelerated flows by extension of the knowledge relating to turbulent boundary layers to conditions where a pressure gradient exists. Calculations for such conditions indicate whether a given velocity distribution and pressure gradient are possible without separation of the boundary layer. A detailed discussion of this is given by L. Prandtl.[8]

C. Certain Concepts Relating to Turbulence

Since the energy dissipation in conduit expansions is closely related to the fluid turbulence, an understanding of some of the basic ideas relating to turbulence is essential. Certain aspects of the statistical theory of turbulence have been presented in a previous paper.[9] The important items relating to turbulence are its origin, designation of its intensity and energy, its diffusing power, and its dissipation.

In general, for various dynamic conditions, regions of high shearing stress, where instability develops, occur in moving fluids, with the result that "rollers" or "eddies" are created. Such regions can occur near solid boundaries in a moving fluid, at boundaries of jets entering a slower-moving or still fluid, and at surfaces between two fluids of different density when a sufficient relative velocity exists. As the vortices or eddies develop, they move away and enter other parts of the fluid. Although the motion of an isolated vortex, such as of the ring or helical type, might be analyzed, when large numbers of such vortices move into a mass of fluid their mutual interactions become so complex

[6] "Pressure Losses Resulting from Changes in Cross-Sectional Area in Air Ducts," by A. P. Kratz and J. R. Fellows, *Bulletin No. 52*, Univ. of Illinois, Urbana, 1938.

[7] "Untersuchungen über die Strömungen des Wassers in konvergenten und divergenten Kanälen," by J. Nikuradse, *Forschungsarbeiten des Vereines Deutscher Ingenieure*, Heft 289, 1929.

[8] "Aerodynamic Theory," W. F. Durand, Editor, Julius Springer, Berlin, Vol. 3, 1935, p. 155 (reprinted by California Inst. Tech.).

[9] "Relation of the Statistical Theory of Turbulence to Hydraulics," by A. A. Kalinske, *Transactions*, Am. Soc. C. E., Vol. 105 (1940), p. 1547.

that it becomes impossible to predict the behavior of a particular eddy. When a mass of fluid has a large number of eddies of varying size and strength, it is said to be turbulent. It has been accepted by various workers in the field of fluid turbulence that the term would be used only to describe fluid motion that is entirely haphazard in character and which can only be described quantitatively in the statistical sense. Thus the well-ordered system of vortices one sees immediately behind a cylinder immersed in a moving fluid stream is not considered as true turbulence.

Intensity and Energy of Turbulence.—The notation to be used for velocity is as follows: The velocity at any instant will be designated by the three rectangular components U, V, and W. The direction of U will be in the direction of mean flow, or, in this paper, along the conduit axis. The components V and W will be at right angles to U. The value of U can be considered as equal to $\overline{U} + u$, in which $\overline{U}$ is the mean axial velocity with respect to time at any point, and u is the fluctuating part due to the turbulence. Since the mean flow is in the direction of $\overline{U}$, $\overline{V}$ and $\overline{W}$ are zero. The turbulence components at any instant perpendicular to the u-component are v and w. Borrowing the concept of "standard deviation" from statistics, the intensity of the turbulence in each of the three directions will be defined as $\sqrt{\overline{u^2}}$, $\sqrt{\overline{v^2}}$, and $\sqrt{\overline{w^2}}$.

The total kinetic energy of the fluid at any point in the section, expressed in foot-pounds per pound of fluid, will then be equal to

$$e = \frac{\overline{U}^2}{2g} + \frac{\overline{u^2} + \overline{v^2} + \overline{w^2}}{2g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The first term is the kinetic energy per pound due to the mean velocity (referred to in hydraulics as velocity head), and will be designated by e_m. The sum of the other terms is the kinetic energy of the turbulence and will be designated by e_t.

An item to keep in mind is that similar values of $\overline{u^2}$, $\overline{v^2}$, and $\overline{w^2}$ for two flow conditions do not necessarily mean that the turbulence is identical. The intensity of the turbulence only characterizes the rotational velocity of the eddies. For a complete description of the turbulence, some measure of the average eddy size is necessary. The diffusing power of the turbulence is directly proportional to both the intensity and the scale.

Turbulence Energy Diffusion and Dissipation.—For a fluid moving under shear, the energy producing the flow is continually being transformed into heat. The work done by the shear, per unit volume of fluid, per unit time, for parallel flow, is equal to $\tau \frac{d\overline{U}}{dy}$, in which τ is the unit shear stress, and $d\overline{U}/dy$ is the mean velocity gradient transverse to the flow. For viscous flow this rate is equal to $\mu \left(\frac{d\overline{U}}{dy}\right)^2$, in which μ is the coefficient of dynamic viscosity. In turbulent flow the rate at which the mean shear does work is not necessarily equivalent to the rate of energy conversion into heat. Part of the work done by the shear results in the creation of turbulence; that is, the energy producing flow is in part transformed into energy of turbulence, which is eventually con-

verted into heat through the action of viscosity. Of course, in the laminar boundary layer of smooth conduits, an appreciable portion of the energy is directly converted into heat since the value of $d\overline{U}/dy$ is very high in this region. Since turbulence is so diffusive in character, the turbulence energy created at one point is not necessarily dissipated into heat at that point.

The dissipation of turbulence energy is an extremely complicated phenomenon from an analytical standpoint because of its three-dimensional character. A general expression for the dissipation of turbulence energy by viscosity has been derived by G. I. Taylor[10] and it contains terms of the type $\overline{(\partial u/\partial x)^2}$, $\overline{\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)^2}$, etc. These terms are the mean of the squares of the instantaneous velocity gradients due to the fluctuating velocity components. For isotropic turbulence, ($\overline{u^2}=\overline{v^2}=\overline{w^2}$), such as that generated in the throat of a wind tunnel due to a regular system of grids, the rate of turbulence energy dissipation, as shown by Mr. Taylor, reduces to a simple expression because of the simple relation that exists between the fluctuating velocity gradients in the different directions. The rate of dissipation per unit volume is then equal to:

$$e_d = 7.5\,\mu\,\overline{\left(\frac{\partial u}{\partial y}\right)^2} \qquad (3)$$

Of course, in conduit flow the turbulence is far from isotropic. The practical significance of Eq. 3 is that it gives a clue as to what factors affect the rate of turbulence energy dissipation. The term $\overline{(\partial u/\partial y)^2}$ can be set proportional to $\overline{u^2}/\lambda^2$, in which λ is a scale of turbulence associated with the dissipation process. Thus, as the size of the eddies decreases, for a given intensity of turbulence, the rate of dissipation increases. Therefore, turbulence can be dissipated more rapidly by breaking up the larger eddies. There is experimental evidence that as turbulence decays, owing to dissipation of its energy, the average scale tends to increase, since the smaller eddies are dissipated more rapidly. The larger eddies are "ground down" in a process that is not as yet clearly understood. There seems to be a stretching of the vortex filaments; for instance, Mr. Taylor[11] shows that the rate of dissipation by viscosity of the vorticity is several times greater than the rate at which vorticity actually disappears. This means that vorticity is being produced through the process of stretching the vortex filaments with a resultant decrease in size of eddies but an increase in the angular velocity, which follows from the law of conservation of angular momentum.

The foregoing ideas will serve as a background for analyzing the data on turbulence that were obtained in the flow through conduit expansions.

D. Experimental and Analytical Procedures

A complete study of the energy conversion and dissipation processes in the flow in expanding conduits involves a study of the mean pressure change

[10] "Statistical Theory of Turbulence," by G. I. Taylor, *Proceedings*, Royal Soc. of London, Vol. 151A, 1935, p. 430.

[11] "Production and Dissipation of Vorticity in a Turbulent Fluid," by G. I. Taylor, *ibid.*, Vol. 164A, 1938, p. 15.

along the conduit, the distributions of mean velocity at various sections, and measurement of the turbulence. So far as the turbulence is concerned, the obtaining of complete information regarding it presents formidable experimental difficulties. It was decided that, if its intensity could be measured at various points in several sections along the conduit, this would be all that could be attempted.

Since this entire investigation extended over several years, changes in regard to general experimental procedures and apparatus occurred as information was obtained relating to experimental difficulties, and also as the general emphasis of the study was altered. The first studies were made on a 15° expansion between pipes of 3-in. and 5-in. diameters. The conical section was constructed of transparent pyralin and the straight pipe of pyralin sheets. Studies on this expansion perfected the experimental techniques, and some useful data were secured. However, after this expansion was in use about a year, it warped and had to be abandoned, thus preventing the collection of a complete set of data for several rates of flow. Since at that time transparent lucite pipe and sheet material had become available, it was decided to complete the studies using various expansions between lucite pipes of 2.75-in. and 4.75-in. diameters. Time permitted obtaining velocity and turbulence data for at least two rates of flow for expansions of 7.5°, 30°, and 180°. Pressure data were obtained on these expansions and also on a 15° expansion between 2.75-in. and 4.75-in. pipe.

The general setup used in the experimental work consisted of a constant-level supply tank, calibrated orifice, approach piping, and regulating valve. After considerable preliminary work, it was decided to obtain the velocity data photographically by use of motion pictures. The technique was as follows: Into the flowing water, about 30 ft upstream from the expansion, a mixture of carbon tetrachloride and benzine was injected, the proportions being adjusted so that the mixture had a density equal to that of the water. Powdered anthracene was suspended in the mixture to give it a milky appearance. The mixture was injected into the flowing water by means of a water-pressure operated injector. The mixture was thus broken up into droplets the size and number of which could be controlled by adjusting the rate of injection and by varying the water pressure on the injector. The size of the particles considered suitable for this work varied between one and two millimeters. The water temperature in these experiments varied from 50° to 65° F.

At sections where velocity data were desired, the flow with the suspended droplets was illuminated with a carbon-arc light by focusing an intense beam through a double-slit barrier, which gave a plane of light about $\frac{1}{8}$ to $\frac{1}{4}$ in. thick. This horizontal plane of light was directed through the center line of the conduit, and particles entering this plane of light were brilliantly illuminated. A 16 mm, motion-picture camera was mounted above the conduit and focused on the plane of light in the conduit. The shutter opening and speed of the camera were adjusted for any run, so that for the given water speed the particles would, on an average, produce streaks about $\frac{1}{2}$ in. long when the film was projected to the actual size of the conduit. A scale was inserted next to the conduit in the field of view to give the linear dimension. The field of view

was only large enough to take in the width of the conduit and the scale. For each run, about 15 ft of motion-picture film was taken, giving about eight hundred individual frames. The time interval for this footage varied with the camera speed, but on the average was about 1.5 min. For each shutter opening and camera speed used, the camera was calibrated for exposure time by photo-

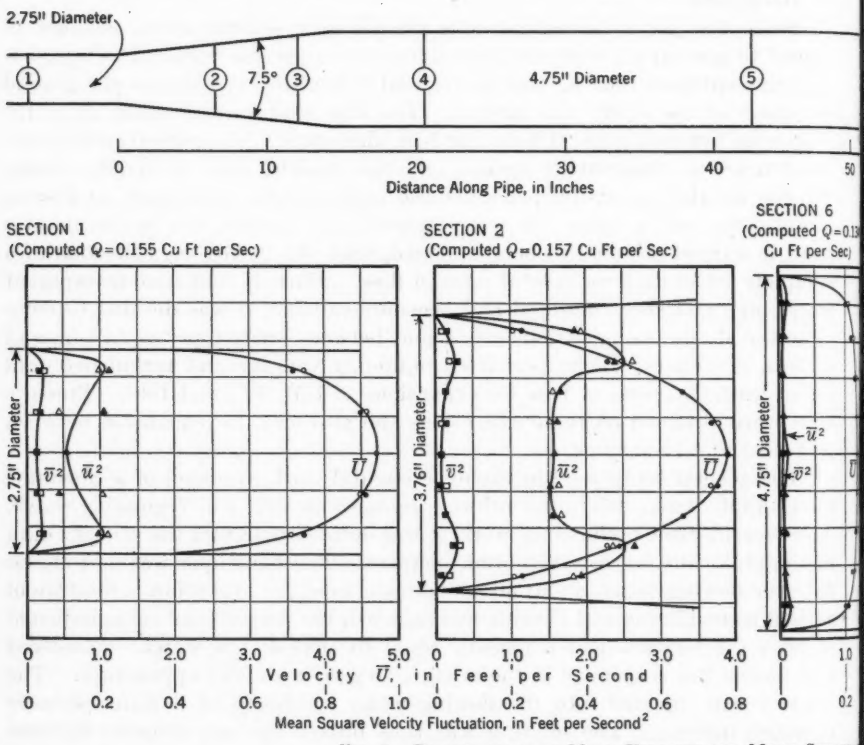
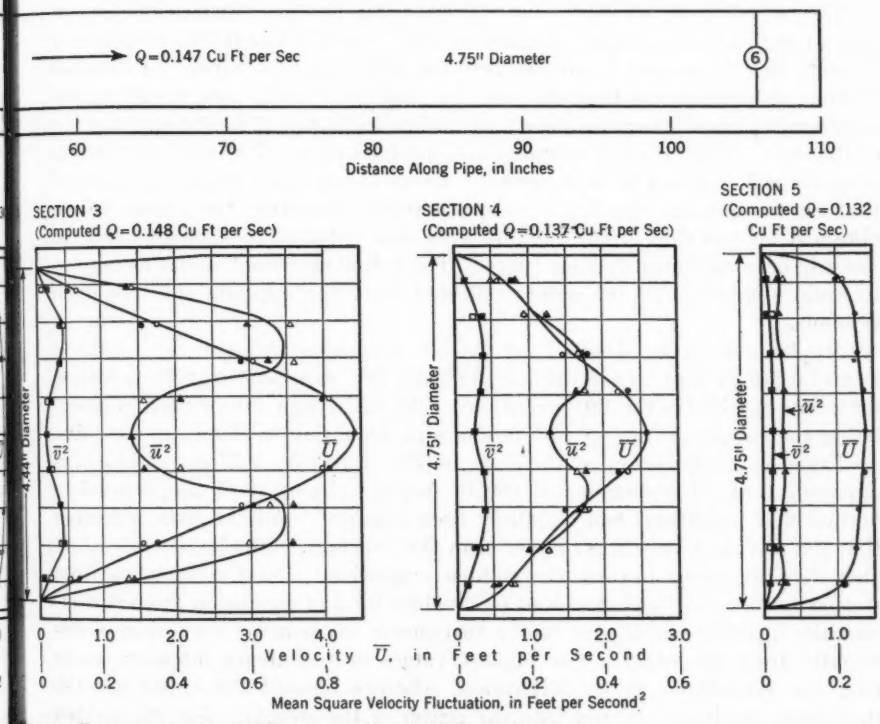


FIG. 1.—DISTRIBUTIONS OF MEAN VELOCITY AND MEAN SQUARE

graphing bright particles attached to a belt which was run by a constant-speed motor.

The pictures were analyzed by projecting the film to somewhat larger than full scale and measuring the length of the streaks in the following manner: At the section of the conduit where velocity data were required, some seven to nine points were selected across the conduit. At these points on the screen, squares were drawn about 1/25th of the conduit diameter in size. Streaks falling on these squares were taken as giving the velocity at that point at that instant, or, more accurately, the average velocity for the instant of time required for the droplet to move the distance of the streak. This time varied with the water speed, but was between 1/20th and 1/80th of a second. Three measurements were taken for each streak—the length components along and perpendicular to the conduit, and the width. The length of the streak in the

direction of the conduit minus the streak width (droplet size) gave the velocity component in that direction at that instant. The average of such lengths gave the mean axial velocity at any point. This is designated by the symbol $\overline{U}$. Subtracting $\overline{U}$ from the individual velocity values at various instances gave u, and from these the value of $\overline{u^2}$ could be obtained.



ELOCITY FLUCTUATIONS FOR 7.5° EXPANSION (Q = 0.147 CU FT PER SEC)

The length component of the streak perpendicular to the conduit gave the velocity component, V, and then, from $(\overline{V} - V)$, values of the turbulent component v could be obtained, and finally $\overline{v^2}$. Thus the streak data permitted the calculation, at each point, of the values of $\overline{U}$, $\overline{u^2}$, and $\overline{v^2}$. The number of motion-picture frames analyzed varied from 500 to 1,000. The number of readings obtained averaged about 300. As can be imagined, the taking of the data from the pictures and their evaluation were very time consuming. However, since such data had never been obtained before, it was thought worth while to do the necessary work, since it seemed that some significant information could be obtained.

The pressure-variation data were obtained by installing wall piezometers along the conduit and connecting these to a tube bank. Colored water was put into the top of the columns so as to make them more visible, and a motion

picture was taken of the water levels in all the tubes. The average value of the pressure was determined by projecting the picture and reading the water-column heights from several hundred frames distributed over a time interval of about 2 min.

E. Mean Velocity and Velocity Fluctuation Data

The distribution of the mean velocity and velocity fluctuations across several sections of a conduit was first obtained for a flow of 0.082 cu ft per sec through the 15° conical expansion between 3-in. and 5-in. pipes. A detailed plotting or tabulation of the values of $\overline{U}$, $\overline{u^2}$, and $\overline{v^2}$ for all the tests would require considerable space; therefore only typical detailed data will be shown for a few of the runs. Figs. 1 and 2 contain data obtained for the 7.5° and 180° expansions for a flow of 0.147 cu ft per sec. From visual observations it appeared that on the average the flow was symmetrical; therefore, the actual values obtained on one side of the conduit were also reflected to the other side to assist in drawing in the average curve. The actual measured values are shown by solid symbols, and the points reflected from the opposite side are open symbols.

Similar data were obtained for the 30° expansion and also for a rate of flow of 0.210 cu ft per sec in the 7.5°, 30°, and 180° expansions, and in addition for a flow of 0.082 in the 180° expansion. All these data will be used in calculating the turbulence energy and in studying its variation along the conduits.

From an examination of the data in Fig. 1 for the 7.5° expansion it is apparent that, at a distance of 100 in. beyond the start of the expansion, normal flow conditions had definitely been attained. All the data indicated that the $\overline{u^2}$-values are always larger than the $\overline{v^2}$-values. The highest $\overline{u^2}$-values and $\overline{v^2}$-values occur beyond the sudden expansion. Relative changes from section to section of $\overline{u^2}$-values and of $\overline{v^2}$-values for any expansion are not comparable; in other words, one of the turbulence components may change differently from the other. The highest values of turbulence intensity occur near the boundaries in the expansion, whereas beyond the expansion the turbulence gradually diffuses into the center of the stream. For the sudden expansion (Fig. 2), backflow near the walls is apparent. It was also observed at the end of the 30° expansion. Only a slight backflow very near the walls was visible for the 15° expansion, and none was noted for the 7.5° expansion.

Values of the discharge were computed by numerical integration of the mean velocity curves and are shown in Figs. 1 and 2. In analyzing the large amount of data it was necessary to use different people at various times, and, since considerable personal error can enter into the readings of the streak lengths from the motion-picture film, it is believed that this explains the discrepancy between the actual and calculated values of Q. In comparing values of $\overline{u^2}$ and $\overline{v^2}$ from section to section, it would be well if these values were corrected by the square of the ratio of calculated discharge to actual discharge. This will be done when the values for the turbulence energy are calculated.

Whether the values of $\overline{u^2}$ and $\overline{v^2}$ that were obtained are correct in absolute magnitude is problematical; however, in any case the relative values should be as indicated. The actual values obtained for normal flow in the straight

2.75" Diameter
4.75" Diameter
Q=0.147 Cu Ft per Sec
Distance Along Pipe, in Inches
SECTION 2 (Computed Q=0.150 Cu Ft per Sec)
SECTION 3 (Computed Q=0.135 Cu Ft per Sec)
SECTION 4 (Computed Q=0.147 Cu Ft per Sec)
SECTION 5 (Computed Q=0.151 Cu Ft per Sec)
4.75" Diameter
Velocity, in Feet per Second
Mean Square Velocity Fluctuation, in Feet per Second²

FIG. 2.—DISTRIBUTIONS OF MEAN VELOCITY AND MEAN SQUARE OF VELOCITY FLUCTUATIONS FOR 180° EXPANSION (Q = 0.147 CU FT PER SEC)

conduits seem to be of the order of magnitude obtained by H. C. H. Townend[12] for flow in a square pipe. Also, as the present data indicate, the value of $\overline{u^2}$ tended to be larger than that of $\overline{v^2}$ in Mr. Townend's studies. He measured the w-component and found it to be of the order of the v-component, except near the wall, where the v-component (perpendicular to the wall) tended toward zero, whereas the w-component did not.

TABLE 1.—MAXIMUM VALUES OF $\sqrt{\overline{u^2}}/\overline{U}_c$

Location of section[a] (see Figs. 1 and 2)	Flow Q, in Cubic Feet per Second: 0.147	0.210	0.082
(*a*) 7.5° Expansion; From 2.75-In. to 4.75-In. Pipe			
0[a]	0.094	0.110	....
6.5	0.181	0.200	....
12.0	0.189	0.197	....
20.5	....	0.322	....
20.6	0.228	....	....
42.7	0.185	0.188	....
64.8	....	0.123	....
105.5	0.112	0.117	....
127.0	....	0.110	....
(*b*) 30° Expansion; From 2.75-In. to 4.75-In. Pipe			
0[a]	0.094	0.110	....
1.6	....	0.228	....
1.8	0.220	....	....
9.9	0.437	0.495	....
17.8	....	0.420	....
22.5	0.278	....	....
64.0	0.142	0.145	....
81.0	....	0.140	....
(*c*) 180° Expansion; From 2.75-In. to 4.75-In. Pipe			
0[a]	0.094	0.110	0.110
7.0	0.231	0.234	0.254
17.0	0.372	0.347	0.284
40.5	0.112	....	0.181
41.0	....	0.150	....
78.8	0.120	....	0.124
79.0	....	0.117	....
(*d*) 15° Expansion; From 3.00-In. to 5-In. Pipe			
0[a]	....	....	0.160
3.0	....	....	0.217
4.0	....	....	0.200
6.0	....	....	0.232
15.5	....	....	0.292
30.5	....	....	0.224

[a] The location of the section is given as the distance in inches from the beginning of the expansion. The values given at zero distance are in the small conduit, although not necessarily just at the end of the small conduit.

Significant items to be noted from Fig. 2 are the manner in which the mean center velocity is reduced and how the turbulence gets into the center stream in the 180° expansion. In the case of the 30° expansion at a distance of 0.5 ft beyond the end of the expansion, the center velocity was reduced from a maximum of 4.7 ft to 2.5 ft per sec. Also, it was noted that at this distance the turbulence created at the boundaries of the expansion had already permeated into the center of the stream. The foregoing reduction in center velocity and permeation of the turbulence do not occur, for the 180° expansion, until a distance of 1.5 ft beyond the expansion (see Fig. 2). Evidently the turbulence created in the 30° expansion is of a larger scale and has greater diffusive power, and thereby penetrates and slows down the center portion of the stream in a shorter distance than in the 180° expansion.

In order to indicate the variation of the relative intensity of the turbulence in the conduits with the various expansions, Table 1 contains values of the ratio of maximum value of $\sqrt{\overline{u^2}}$ to the center velocity in the cross section, $\overline{U}_c$. Data shown in this table are for all the measurements made in these experiments. It is noted that the maximum value of this ratio occurs for the 30° expansion and is about 0.50 for the flow of 0.210 cu ft per sec. This ratio seems to have

[12] "Statistical Measurements of Turbulence in the Flow of Air Through a Pipe," by H. C. H. Townend, *Proceedings*, Royal Soc. of London, Vol. 145A, 1934, p. 180.

about the same value in the 2.75-in. pipe and in the 4.75-in. pipe, when normal flow is again developed. It seems that it took a distance of about 125 in. or about twenty-five diameters from the start of the expansions for this ratio to attain a constant value in the large pipe. For any expansion and at any given section this ratio, in general, has a smaller value for the smaller rate of flow.

An examination of the typical detailed data shown in Figs. 1 and 2 and the values in Table 1 offers a fairly complete picture of the variation of the turbulence in conduit expansions.

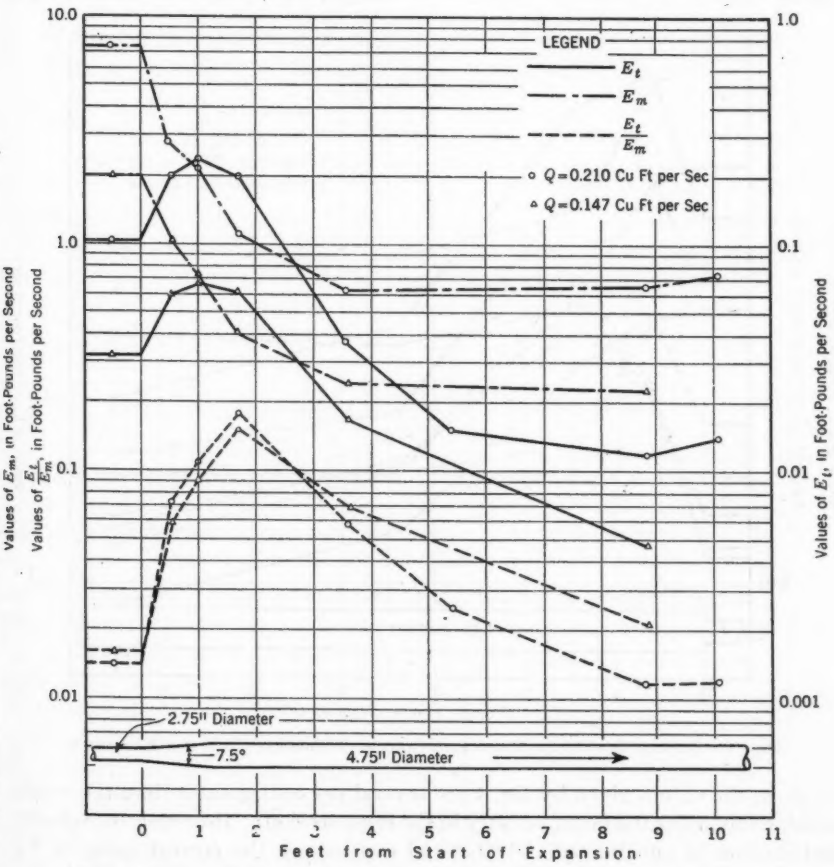


FIG. 3.—KINETIC ENERGY OF MEAN FLOW AND OF TURBULENCE FOR 7.5° EXPANSION

F. MEAN KINETIC ENERGY AND TURBULENCE ENERGY

To indicate the changes occurring in the kinetic energy of the mean flow and of the turbulence energy along the conduit for the several expansions, the total value of these energies was calculated by numerical integration of data such as those shown in Figs. 1 and 2. The total kinetic energy of the mean

flow in foot-pounds per second was obtained from the following expression:

$$E_m = \frac{\pi \gamma}{g} \int_0^r \overline{U}^3 y \, dy \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which γ is the weight of water per unit volume, r is the radius of the pipe section, and y is the distance from the center. If the distribution of the mean velocity was uniform, E_m could be expressed as $\frac{Q \gamma \overline{U}^2{}_m}{2g}$, in which $\overline{U}_m$ is the mean velocity in a cross section. For normal turbulent flow in a circular

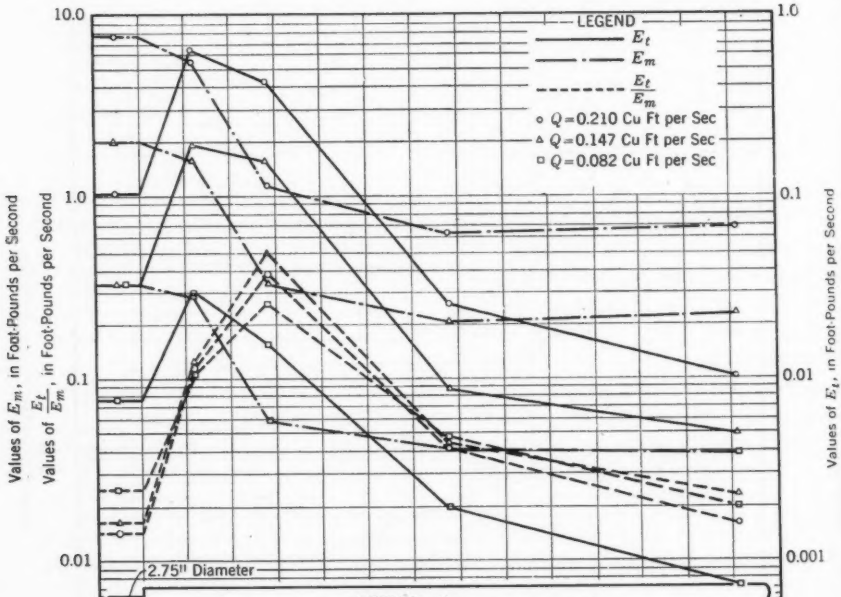


FIG. 4.—KINETIC ENERGY OF MEAN FLOW AND OF TURBULENCE FOR 180° EXPANSION

conduit, the values given by Eq. 4 are several per cent greater than the values obtained by using the mean velocity in the cross section. However, for velocity distribution in and immediately beyond expansions, the correct value of E_m may be several times the value obtained by using the mean velocity in the cross section. The total kinetic energy of the turbulence per pound of fluid flowing is equal to

$$e_t = \frac{\overline{u^2} + \overline{v^2} + \overline{w^2}}{2g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The total kinetic energy of the turbulence in the water flowing per unit of time

past any section is then given by:

$$E_t = \frac{\pi \gamma}{g} \int_0^r \overline{U}(\overline{u^2} + \overline{v^2} + \overline{w^2})\, y\, dy \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

In the present studies the value of $\overline{w^2}$ was not determined; however, in the evaluation of E_t, $\overline{w^2}$ was taken equal to $\overline{v^2}$. This appears to be a fairly good approximation except near the walls of the conduit. The values of E_t were obtained by numerical integration of data such as those shown in Figs. 1 and 2.

The ratio of calculated to actual discharge at any section was assumed to indicate the error in the streak length or in the length of velocity scale used. The calculated values of E_m or E_t were therefore corrected by the cube of this ratio. In Figs. 3 and 4 and in Table 2 are shown the values of E_m, E_t, and the ratio E_t/E_m for all the data obtained in these studies. Several pertinent items might be mentioned regarding these data: The maximum value of E_t occurs right at the end of the 7.5° expansion; for the 15° expansion it occurs slightly beyond; and, for the 30° and 180° expansions, the maximum value of E_t is about 0.60 ft beyond the expansion. The maximum value of the ratios E_t/E_m always occurs beyond the end of the expansions; the distances beyond the various expansions were 1.4 ft for 180°, 0.65 ft for 30°, and 0.50 ft for 7.5°. The turbulence energy appears to have ceased changing in the 4.75-in. conduit at a distance of 9 ft or about twenty-three diameters from the start of the 7.5° expansion. However, the change in E_m is fairly small beyond about ten diameters from the start of the expansions.

TABLE 2.—KINETIC ENERGY OF MEAN FLOW AND OF TURBULENCE

Location of section[a] (see Figs. 2 and 3)	KINETIC ENERGY, IN FOOT-POUNDS PER SECOND		Ratio, E_t/E_m
	Mean flow, E_m	Turbulence, E_t	
(a) 15° EXPANSION FROM 3-IN. TO 5-IN. PIPE; Q = 0.082 CU FT PER SEC			
0[a]	0.221	0.0067	0.030
0.25	0.157	0.0106	0.068
0.33	0.125	0.0110	0.088
0.50	0.101	0.0122	0.121
1.30	0.042	0.0122	0.291
2.55	0.038	0.0042	0.110
(b) 30° EXPANSION FROM 2.75-IN. TO 4.75-IN. PIPE; Q = 0.147 CU FT PER SEC			
0[a]	2.00	0.032	0.016
0.15	1.42	0.110	0.078
0.85	0.481	0.230	0.480
1.90	0.221	0.040	0.182
5.35	0.220	0.006	0.027
(c) 30° EXPANSION FROM 2.75-IN. TO 4.75-IN. PIPE; Q = 0.210 CU FT PER SEC			
0[a]	7.56	0.101	0.014
0.15	3.85	0.382	0.099
0.85	1.40	0.781	0.513
1.40	0.765	0.248	0.325
5.35	0.622	0.031	0.050
6.75	0.570	0.013	0.023

[a] The location of the section is given as the distance, in feet, from the beginning of the expansion. Values given at zero distance are in the small conduit.

The approximate maximum values of E_t/E_m for the various expansions tested are as follows: 7.5°, 0.17; 15°, 0.28; 30° and 180°, 0.50. The values appear to be somewhat higher for the lower rates of flow. The actual values of E_t for the 30° expansion are slightly greater than those for the 180° expansion. The rate of decrease of E_t beyond the expansion is apparently most rapid for the 180° expansion, indicating that the turbulence is probably finer grained for the 180° than for the 30° expansion and thus dissipates faster. This was mentioned in Section E in connection with another item. The fact

that the 30° expansion appears to have as much total turbulence energy as the 180° expansion does not necessarily mean that the head loss through it is as great. The intensity of the turbulence at a point in the fluid corresponds somewhat to the magnitude of the friction or shear stress. If this intensity is high, even if it is quite localized, the energy loss due to friction is certain to be large. It was noted that the maximum intensity of the turbulence for the 180° expansion is about 1.3 times greater than that for the 30° expansion for any given rate of flow.

TABLE 3.—PRESSURE VARIATION ALONG CONDUITS, IN FEET OF WATER
(Expansion from 2.75-In. Pipe to 4.75-In. Pipe)

Location of section[a]	$Q = 0.147$	$Q = 0.210$
(*a*) 15° EXPANSION		
−1.50	0.100	0.100
0.00	0.080	0.051
0.35	0.162	0.215
0.75	0.177	0.250
0.90	0.186	0.265
1.20	0.202	0.285
1.92	0.208	0.308
3.35	0.212	0.314
7.20	0.202	0.305
(*b*) 30° EXPANSION		
−1.50	0.100	0.100
0.00	0.080	0.051
0.42	0.131	0.175
0.60	0.150	0.210
0.86	0.172	0.250
1.62	0.191	0.288
3.05	0.194	0.294
6.92	0.185	0.282

[a] The location of the section is given as the distance in feet from the beginning of the expansion (see Fig. 5). The flow Q is in cubic feet per second.

The aforementioned items seem to be the principal information that can be obtained from an examination of the E_m-curves and E_t-curves. These quantitative data give some information relating to the changes in the mean kinetic and turbulence energy that take place in the flow through expanding conduits.

G. PRESSURE DATA

To complete the picture of the energy conversion process for flow through expanding conduits it is necessary to have data on the variation of the pressure. Pressures were measured for the 7.5°, 15°, 30°, and 180° expansions in the manner described in Section *D*. The pressure variation data for these expansions are shown in Fig. 5 and Table 3.

The very appreciable drop in the wall pressure at the start of the larger conduit for the sudden expansion, as shown in Fig. 5(*b*), is quite characteristic of this type of expansion. The maximum pressure is reached at distances beyond the start of the expansions which seem to increase with decrease in angle; thus, for the 7.5° expansion this distance is about 4.0 ft, for 15° it is 3.3 ft, for 30° it is 2.8 ft, and for 180° it is 2.5 ft.

In order to indicate the change in the total energy, exclusive of the turbulence energy, along the conduits, the quantity $(E_p + E_m)$ was calculated and is plotted in Fig. 6. The value of E_p was obtained by taking the values of pressure head, h, and multiplying by $62.4\,Q$. It is worth noting that the change that occurs in this total energy in the 180° expansion is much more gradual than that in the 30° expansion. In other words, as was mentioned previously, the energy conversion occurs more gradually in the sudden expansion than in the 30° expansion due to the fact that the relatively small-scale turbulence that is generated at the edges of the high velocity stream is not so effective in "eating into" the stream and slowing it down. The diffusive power of the turbulence depends on both its intensity and scale and evidently

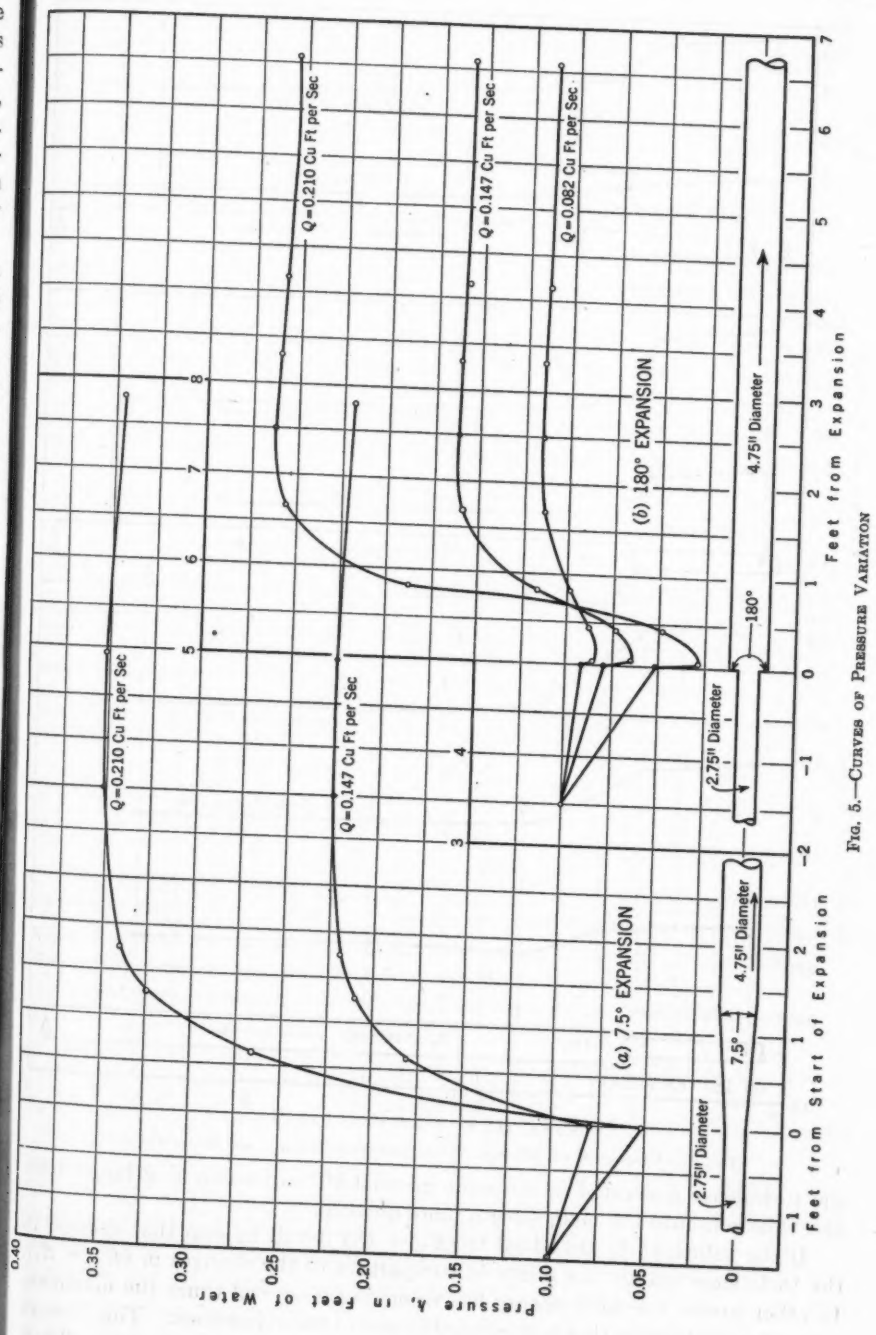


FIG. 5.—CURVES OF PRESSURE VARIATION

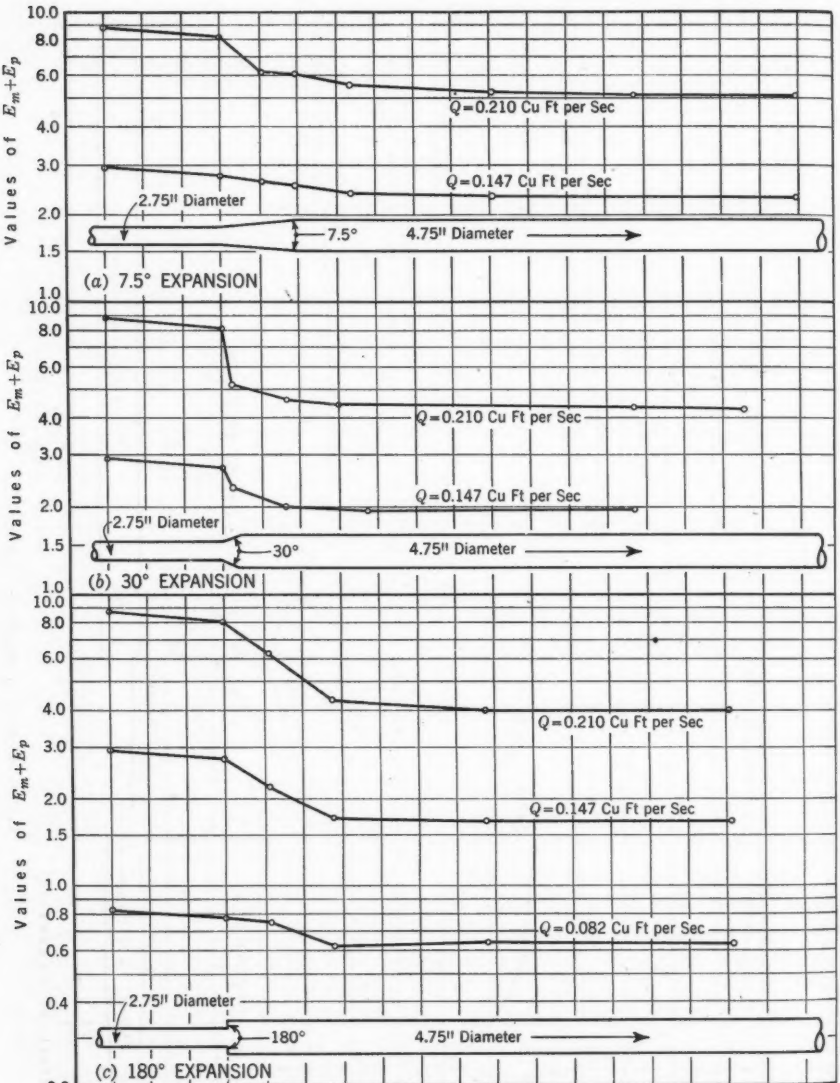


FIG. 6.—VARIATION OF KINETIC PLUS POTENTIAL ENERGY OF MEAN FLOW

the turbulence generated in the more gradual 30° expansion is of larger scale and thus gets into the main stream more quickly.

If the values of E_t are added to $(E_m + E_p)$ it will be seen that changes in the turbulence energy are minor in comparison to the changes in $(E_m + E_p)$. In other words, the total energy loss occurring is several times the maximum amount of turbulence that is generated in any of the expansions. This appears to be one of the most significant facts coming out of these studies. What,

then, is the exact rôle played by the turbulence? It appears that the major portion of the kinetic energy of mean flow is lost directly due to the locally high, and quite variable, shear stresses that are set up in the fluid on the outside of the high-velocity jet. The turbulence created is a kind of by-product which, of course, uses up some of the kinetic energy of the mean flow. Although a by-product of the major energy-loss process, the turbulence, none the less, has an important influence on slowing down the high-velocity water and in shaping the velocity distribution curve.

From the values of $(E_m + E_p)$ it was possible to calculate the actual loss of energy. This was done by taking the difference between the values of $(E_m + E_p)$ at the start of the expansion and at about twenty to thirty diameters beyond the expansion and subtracting from this the normal friction in the conduit between the two points. In the conical expansions the friction loss was calculated by summing the loss for increments along the expansion. The values of the energy loss, expressed in feet of water, are shown in Table 4. Also, the efficiency of the energy conversion is given, which is expressed as:

$$\eta = \frac{(h_2 - h_1)\, Q\, \gamma}{E_{m1} - E_{m2}} \qquad (7)$$

in which $(h_2 - h_1)$ is the rise in pressure head (after correction for friction loss), and the denominator is the difference in the mean kinetic energy.

TABLE 4.—ENERGY LOSS, h, AND EFFICIENCY, η, OF VARIOUS EXPANSIONS FROM 2.75-IN. PIPE TO 4.75-IN. PIPE

Flow (cu ft per sec)	7.5° EXPANSION		15° EXPANSION		30° EXPANSION		180° EXPANSION	
	h (ft)	η (%)	h (ft)	η (%)	h (ft)	η (%)	h (ft)	η (%)
0.082	....	....	....	....	....	....	0.027	51
0.147	0.037	81	0.058	70	0.081	59	0.108	45
0.210	0.211	60	0.238	54	0.277	48	0.296	43

The energy loss values, as given in Table 4, do not appear to vary consistently with rate of flow. Checking the loss figures given for the 180° expansion with the values obtained from the Borda formula indicates that for the higher rates of flow the figures given are appreciably larger. The reason for these apparent discrepancies is that the kinetic energy of the mean flow in the small pipe is considerably greater that that which would be obtained using the mean velocity in the cross section. For instance, for the flow of 0.210 cu ft per sec, the ratio of the actual to approximate total kinetic energy in the 2.75-in. pipe is 1.42; for the flow of 0.147 cu ft per sec, it is 1.11; and, for 0.082 cu ft per sec, it is 1.02. The variation in this ratio accounts for the fact that the loss does not vary as the square of the rate of flow. For 0.082 cu ft per sec, the loss in the sudden expansion checks that obtained from the Borda equation very well, and the reason is apparent. It is thus obvious that the energy loss in an expansion will depend on the velocity distribution at the inlet side, and this distribution must be taken into account if the actual loss is to be correctly evaluated. It is noted that the efficiency of energy conversion in the 30° expansion is not much better than in the 180° expansion.

H. Conclusions

Analysis of experimental data obtained in this investigation leads to the following conclusions relating to the energy conversion and dissipation processes in circular pipe expansions:

1. The largest energy changes occurring in the flow through any expansion are those due to the reduction in velocity and change in shape of the mean velocity distribution curve.

2. The maximum total turbulence energy is a small part of the total energy change taking place. However, at any particular section the turbulence energy may be a considerable part of the total kinetic energy. For the 30° expansion and 180° expansion, the maximum value of the ratio of turbulence energy to mean kinetic energy, E_t/E_m, rises to 0.50. For the 7.5° expansion it is only 0.17.

3. The maximum ratio between the root-mean-square value of the axial velocity fluctuation, $\overline{u^2}$, and the mean center velocity occurs in the 30° expansion, and is 0.50. The transverse velocity fluctuation is always appreciably less than the axial.

4. The rate of loss of energy occurs more rapidly in the 30° expansion than in the sudden expansion.

5. The principal loss of energy occurs at the outside of the high-velocity stream as it passes into the expansion due to the very high shear developed. This loss is represented by a direct conversion into heat of the mean kinetic energy of the flow. The turbulence produced is but a by-product of this energy conversion process. Of course, the turbulence created also represents lost energy, since eventually it is dissipated into heat.

6. The turbulence created in the expansion plays an important rôle in helping to reduce the velocity of the center portion of the stream. Larger scale turbulence is more effective in this than in fine-grained turbulence.

7. The energy lost in an expansion and the efficiency of the conversion process are influenced considerably by the distribution of the mean velocity at the inlet to the expansion. High ratios of center velocity to the mean velocity in the cross section produce poor efficiencies and large energy losses.

I. Acknowledgments

Since the experimental work reported herein extended over a period of six years, it is natural that many individuals should have contributed their efforts. This project was sponsored by the Hydraulic Research Committee of the Society, and financial aid was provided by the Engineering Foundation. The work was done in the hydraulics laboratory of the Iowa Institute of Hydraulic Research.

F. T. Mavis, M. Am. Soc. C. E., formerly head of the Department of Mechanics and Hydraulics at the University of Iowa, at Iowa City, is responsible for starting this investigation, and for his advice, suggestions, and encouragement the writer is very grateful. Graduate students who assisted at various times with the laboratory and analytical work were: E. R. Van Driest and C. L. Pien, Assoc. Members, Am. Soc. C. E., A. R. Luecker, J. M. Robertson, and Claude Lomax, Jr., Juniors, Am. Soc. C. E., and W. M. Wachter. The National Youth Administration provided a considerable number of assistants for the statistical work.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

COLUMN FORMULAS

By William R. Osgood,[1] M. Am. Soc. C. E.

Synopsis

An empirical column formula should be one that expresses theoretically possible relations. The prime requisites of any column formula or formulas are that the average stress be a continuous, monotonically decreasing function of the ratio of slenderness and that it approach the Euler value as the ratio of slenderness becomes infinite. Another desirable requisite is that the first derivative of the average stress with respect to the ratio of slenderness be continuous. These requirements impose certain restrictions on the constants in empirical formulas, and these restrictions are considered for a number of well-known and less well-known column formulas.

Introduction

Column formulas are a convenience, not a necessity, for relating the strength of a column to its dimensions. It would be quite possible to plot the results of tests, determine a curve from the plot, and select design values from this curve or from a table of values read from the curve. When a formula is used, however, as it usually is, there is some virtue in having it of correct form, if only because the designer may have to extrapolate beyond the values on which the formula is based. If, for example, at large ratios of slenderness a formula covering all ratios of slenderness leads to values of average stress that are known to be impossible, the formula will lead at less large ratios of slenderness to values that are likely to be impossible. Therefore it is desirable not to preclude extrapolation. Is it not at least slightly better to know that extrapolated values may be right than to know that they probably are wrong?

There have been numerous and elaborate attempts to devise formulas based on elastic theory for the design of centrally loaded columns in the plastic range. Before the development of the double-modulus theory toward the end of the past century these attempts were consonant with good engineering practice. Today, however, it should be realized how futile such attempts must be. It

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by May 1, 1945.

[1] Materials Engr., National Bureau of Standards, Washington, D. C.

would be possible to design by means of the double-modulus theory, given the compressive stress-strain curve of the material; but the best testing technique results in wider discrepancies between theoretical and experimental values in the plastic range than in the elastic range, and most engineers would undoubtedly prefer to use an empirical formula based on tests. An empirical formula, however, should be one that expresses theoretically possible relations. Besides being of correct form, the formula should contain separately one constant of the structure, implicitly or explicitly, and one or more constants related to the material. It would be very hard to justify more than two constants of the material in any practical column formula.

As to the form, the prime requisites of any column formula or formulas are that the average stress be a continuous, monotonically decreasing function of the ratio of slenderness and that it approach the Euler value as the ratio of slenderness becomes infinite. Another desirable requisite is that the first derivative of the average stress with respect to the ratio of slenderness be continuous.

It is proposed to present a number of formulas in their most general forms to satisfy the requirements of the preceding paragraph. For comparative purposes the formulas will all be written in terms of the two non-dimensional variables, λ_o (the reduced ratio of slenderness) and σ (the reduced average stress):

$$\lambda_o = \frac{1}{\pi}\frac{l_o}{i}\sqrt{\frac{S}{E}} \qquad (1a)$$

and

$$\sigma = \frac{P}{A\,S} \qquad (1b)$$

as well as in terms of the customary variables—the average stress P/A, and the ratio of slenderness l_o/i. The free length is:

$$l_o = k\,l \qquad (2)$$

l being the actual length (panel-point length) of the column and k being the constant of the structure—namely, the factor depending on end conditions by which l must be reduced or increased to obtain the distance, l_o, between successive points of inflection. For simply supported ends, $k = 1$; for fixed ends, $k = \frac{1}{2}$; for one end free and the other end fixed, $k = 2$; etc. In Eqs. 1, E is the modulus of elasticity of the material, and S is an arbitrary constant of the dimensions of stress. In analyzing column-test data, S should be taken as a compressive secant yield strength. (The secant yield strength is the stress at the intersection with the stress-strain curve of a line through the origin having a slope $m\,E$, where $0 < m < 1$.)

The most general forms of some well-known and other, less well-known formulas will now be given. The dimensionless constants β, γ, and n, and the constants B and C that appear are arbitrary, subject to the conditions stated in each case. They are constants related to the material. The load, P, is to be considered the maximum load, and after assigning values to the constants, P is to be taken as the working load multiplied by a factor of safety.

The only formula containing but one arbitrary constant that will be considered is the Rankine formula. This formula is used extensively. The justification for it is that at low and at high ratios of slenderness it fits test results well, and at intermediate ratios of slenderness, where the scatter in test results is relatively large, the formula gives lower values of the average stress than the mean of the average stresses found in tests of most materials. The formula applies for all values of ratio of slenderness.

I. The Rankine Formula

The Rankine formula is expressed by

$$\sigma = \frac{1}{\beta + \lambda^2_o}; \qquad \beta \geqq 0 \ldots\ldots\ldots\ldots\ldots\ldots (3a)$$

or

$$\frac{P}{A} = \frac{\pi^2 E}{B + \left(\frac{l_o}{i}\right)^2}; \qquad B \geqq 0 \ldots\ldots\ldots\ldots\ldots\ldots (3b)$$

When $\beta = B = 0$, Eqs. 3 reduce to the Euler formula,

$$\sigma = \frac{1}{\lambda^2_o} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4a)$$

or

$$\frac{P}{A} = \frac{\pi^2 E}{\left(\frac{l_o}{i}\right)^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4b)$$

It is easy to determine β from a series of observed values of σ and λ_o by plotting $\sigma \lambda^2_o$ against σ, for

$$\sigma \lambda^2_o = -\beta \sigma + 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

and $-\beta$ is the slope of the resulting straight line (if one results!) through the point $\sigma = 0$, $\sigma \lambda^2_o = 1$. Alternatively, since

$$\beta = \frac{1}{\sigma} - \lambda^2_o \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

β may be taken as the average of the values of $\frac{1}{\sigma} - \lambda^2_o$.

There are several formulas containing two arbitrary constants, and one of them, and often more than one, can be made to fit almost any column-test data that it may be desired to fit. First will be considered formulas covering the entire range of ratios of slenderness.

II. The Secant Formula

The secant formula is expressed by:

$$\lambda_o = \frac{2}{\pi \sqrt{\sigma}} \sec^{-1}\left(\frac{1}{\beta \sigma} + C\right); \qquad \beta \geqq 0, \qquad C \leqq 1 \ldots\ldots\ldots (7a)$$

or

$$\frac{l_o}{i} = \frac{2\sqrt{E}}{\sqrt{\frac{P}{A}}} \sec^{-1}\left(\frac{1}{B\frac{P}{A}} + C\right); \qquad B \geqq 0, \qquad C \leqq 1 \ldots\ldots\ldots (7b)$$

When $\beta = B = 0$, Eqs. 7 reduce to Euler's formula.

The constants β and C may be determined by plotting $\sec \frac{\pi}{2}\lambda_o \sqrt{\sigma}$ against $1/\sigma$, for

$$\sec \frac{\pi}{2}\lambda_o \sqrt{\sigma} = \frac{1}{\beta\sigma} + C \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

and $\frac{1}{\beta}$ and C are the slope and intercept, respectively, of what may turn out to be a straight line.

III. Aarflot's Formula

A column formula due to M. G. Aarflot[2] is:

$$\sigma = \frac{1}{C\lambda^2_o + (1 - C)\sqrt{\lambda^4_o + \beta}}; \qquad \beta \geqq 0, \qquad 0 \leqq C \leqq 1 \ldots\ldots (9a)$$

or

$$\frac{P}{A} = \frac{\pi^2 E}{C\left(\frac{l_o}{i}\right)^2 + (1 - C)\sqrt{\left(\frac{l_o}{i}\right)^4 + B}}; \qquad B \geqq 0, \qquad 0 \leqq C \leqq 1 \ldots (9b)$$

When $\beta = B = 0$, or $C = 1$, Eqs. 9 reduce to Euler's formula.

The constants β and C may be determined from a series of observed values of σ and λ_o by plotting $\frac{1}{\sigma^2} - \lambda^4_o$ against $\lambda^2_o\left(\frac{1}{\sigma} - \lambda^2_o\right)$, for

$$\frac{1}{\sigma^2} - \lambda^4_o = 2C\lambda^2_o\left(\frac{1}{\sigma} - \lambda^2_o\right) + \beta(1 - C)^2 \ldots\ldots\ldots\ldots (10)$$

and $2C$ and $\beta(1 - C)^2$ are the slope and intercept, respectively, of the straight line represented by Eq. 10.

IV. Ylinen's Formula

A column formula due to A. A. J. Ylinen[3] is:

$$\lambda_o = \sqrt{\frac{1 - \beta\sigma^n}{\sigma}}; \qquad \beta \geqq 0, \qquad n > 0 \ldots\ldots\ldots\ldots (11a)$$

---

[2] "A New Column Formula," by Martin G. Aarflot, *Bulletin*, Am. Soc. of Swedish Engrs., Vol. 26. February, 1931, p. 7.

[3] "Keskeisesti Puristetun Sauvan Nurjahtamisesta Suhteellisuusrajan Yläpuolella," by Arvo A. J. Ylinen, *Teknillinen Aikakauslehti*, Vol. 21, 1931, p. 95.

or

$$\frac{l_o}{i} = \pi \sqrt{E} \sqrt{\frac{1 - B\left(\frac{P}{A}\right)^n}{\frac{P}{A}}}; \qquad B \geqq 0, \qquad n > 0 \ldots\ldots\ldots (11b)$$

When $\beta = B = 0$, Eqs. 11 reduce to Euler's formula, and when $\beta > 0$, $B > 0$, and $n = 1$, they reduce to Rankine's formula.

It is easy to determine whether the formula is a good fit to test data and to determine the constants if it is, for

$$\log (1 - \lambda^2_o \sigma) = n \log \sigma + \log \beta \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

and it is only necessary to plot observed values of $1 - \lambda^2_o \sigma$ against σ on logarithmic paper. If the plotted points fall on a straight line, the constants may be obtained from the slope and intercepts of this line.

V. Formulas Proposed by the Writer

A formula proposed by the writer is:

$$\sigma = \frac{1}{\beta + \gamma \lambda_o + \lambda^2_o}; \qquad \beta \geqq 0, \qquad \gamma \geqq 0 \ldots\ldots\ldots\ldots (13a)$$

or

$$\frac{P}{A} = \frac{\pi^2 E}{B + C\frac{l_o}{i} + \left(\frac{l_o}{i}\right)^2}; \qquad B \geqq 0, \qquad C \geqq 0 \ldots\ldots\ldots (13b)$$

When $\beta = \gamma = B = C = 0$, Eqs. 13 reduce to Euler's formula, and, when $\beta > 0$, $B > 0$, and $\gamma = C = 0$, they reduce to Rankine's formula.

The constants β and γ may be determined by plotting $\frac{1}{\sigma} - \lambda^2_o$ against λ_o, for

$$\frac{1}{\sigma} - \lambda^2_o = \gamma \lambda_o + \beta \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

and, if the resulting plot is approximately a straight line, γ and β are its slope and intercept, respectively. If a straight line does not result, Eqs. 13 are inapplicable. (Eqs. 13 may not be new, and the writer would appreciate having a reference to them if any reader knows of one.)

Another suggestion by the writer is:

$$\lambda_o = \frac{1}{\sqrt{\sigma + \beta \sigma^n}}; \qquad \beta \geqq 0, \qquad n > 1 \ldots\ldots\ldots\ldots (15a)$$

or

$$\frac{l_o}{i} = \frac{\pi \sqrt{E}}{\sqrt{\frac{P}{A} + B\left(\frac{P}{A}\right)^n}}; \qquad B \geqq 0, \qquad n > 1 \ldots\ldots\ldots (15b)$$

When $\beta = B = 0$, Eqs. 15 reduce to Euler's formula. As $n \to \infty$, Eq. 15*a* approaches a broken curve consisting of the axis of σ down to $\sigma = 1$, the line $\sigma = 1$ between $\lambda_o = 0$ and $\lambda_o = 1$, and the reduced Euler curve beyond $\lambda_o = 1$.

As in the case of Eqs. 11 it is easy to determine whether Eq. 15*a* represents a good fit to test data, and to determine the constants if it does, for

$$\log\left(\frac{1}{\lambda^2_o} - \sigma\right) = n \log \sigma + \log \beta \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

Eqs. 15 are derived by the double-modulus theory from a stress-strain curve having the equation

$$e = \frac{P}{A\,E} + K\left(\frac{P}{A\,E}\right)^n \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

in which K and n are empirical constants.

A third suggestion by the writer is:

$$\lambda_o = \frac{2}{\sqrt{\sigma} + \sqrt{\sigma + \beta\,\sigma^n}}; \qquad \beta \geqq 0, \qquad n > 1 \ldots\ldots\ldots (18a)$$

or

$$\frac{l_o}{i} = \frac{2\,\pi\sqrt{E}}{\sqrt{\frac{P}{A}} + \sqrt{\frac{P}{A} + B\left(\frac{P}{A}\right)^n}}; \qquad B \geqq 0, \qquad n > 1 \ldots\ldots\ldots (18b)$$

When $\beta = B = 0$, Eqs. 18 reduce to Euler's formula. Like Eq. 15*a*, as $n \to \infty$, Eq. 18*a* approaches a broken curve consisting of the axis of σ down to $\sigma = 1$, the line $\sigma = 1$ between $\lambda_o = 0$ and $\lambda_o = 1$, and the reduced Euler curve beyond $\lambda_o = 1$. Also, like Eqs. 15, Eqs. 18 are derived by the double-modulus theory from a stress-strain curve of the form of Eq. 17.

As in the case of Eqs. 11 the constants in Eqs. 18 may be determined easily and accurately, for

$$\log \frac{4}{\lambda_o}\left(\frac{1}{\lambda_o} - \sqrt{\sigma}\right) = n \log \sigma + \log \beta \ldots\ldots\ldots\ldots\ldots (19)$$

The next two formulas (Eqs. 20 and 21) contain two arbitrary constants each, but the formulas are intended for use only above the limiting average stresses indicated. The curves they represent are tangent to the Euler curve, and for stresses below the point of tangency the Euler formula, Eqs. 4, should be used.

VI. The Parabolic-Type or Hyperbolic-Type Formula

The following formula is of a parabolic or a hyperbolic type:[4]

$$\sigma = \beta\left[1 - \frac{2}{n+2}\left(\frac{n\,\beta}{n+2}\right)^{n/2}\lambda^n_o\right];$$

$$\beta > 0, \qquad n > 0, \qquad \sigma \geqq \frac{n\,\beta}{n+2} \ldots\ldots\ldots\ldots\ldots (20a)$$

[4] "Column Curves and Stress-strain Diagrams," by William R. Osgood, *Journal of Research*, National Bureau of Standards, Vol. 9, October, 1932, p. 571.

or

$$\frac{P}{A} = B\left\{1 - \frac{2}{n+2}\left[\frac{n\,B}{(n+2)\,E}\right]^{n/2}\frac{1}{\pi^n}\left(\frac{l_o}{i}\right)^n\right\};$$

$$B > 0, \qquad n > 0, \qquad \frac{P}{A} \geqq \frac{n\,B}{n+2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (20b)$$

When $n = 1$, Eqs. 20 become the straight-line formula; and when $n = 2$, they become the Johnson parabolic formula.

VII. Kreüger's Formula

A formula due to H. Kreüger[5] is

$$\sigma = \frac{\beta - \lambda^2_o}{\frac{1}{4}(\beta - \gamma)^2 + \gamma\,\lambda^2_o}; \qquad \beta > 0, \qquad -\beta \leqq \gamma \leqq \beta, \qquad \sigma \geqq \frac{2}{\beta - \gamma} \ldots (21a)$$

or

$$\frac{P}{A} = \frac{\pi^2\,E\,B - \left(\frac{l_o}{i}\right)^2}{\frac{1}{4}\,\pi^2\,E\,(B - C)^2 + C\left(\frac{l_o}{i}\right)^2};$$

$$B > 0, \qquad -B \leqq C \leqq B, \qquad \frac{P}{A} \geqq \frac{2}{B - C} \qquad \ldots\ldots\ldots\ldots (21b)$$

When $\gamma = C = 0$, Eqs. 20 reduce to the Johnson parabola.

Summary

There is little to be said for a formula on the grounds of its simplicity, for any formula can be plotted to as large a scale as desired and values read from the graph, or it may be tabulated with increments in the argument as small as desired. On the other hand, there is no point in using a formula that is applicable only down to a given value of the average stress, or up to a given value of the ratio of slenderness, beyond which the Euler formula must be used. The column formulas in Sections II to V, inclusive, apply for all values of the ratio of slenderness, and one or all of them can almost be guaranteed to fit adequately any reasonable column-test data it may be desired to fit.

[5] "Bidrag till fragån om sträfvors knäckning," by H. Kreüger, *Teknisk Tidskrift*, Väg- och Vattenbyggnadskonst, Vol. 45, September 15, 1915, p. 101.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

RIGIDITY AND AERODYNAMIC STABILITY OF SUSPENSION BRIDGES

Discussion

BY LOUIS BALOG

LOUIS BALOG,[74] ESQ.[74a]—The basic formula of this paper is Eq. 6, representing the stiffness of suspension bridges by the summation of expressions that characterize the stiffness of the component parts—cable and girder—of the center span. The cable is assumed to be a string under tension, rigidly held at its ends; the girder, to be a simple beam of uniform section. The accuracy of this approximation when determining the frequencies of a practically unstiffened bridge is well indicated by the agreement between the calculated and observed values for the Tacoma Narrows Bridge. However, the equations given for the condition of three spans coupled do not define the values of n_1 corresponding to values of n in Table 1. For bridges with stiffening girders of small rigidity, the exceedingly simple Eq. 17, which was presented by Joseph Melan in 1906,[75] gives useful results. The frequencies of the Tacoma Narrows Bridge differed by only 6% up to the observed $n = 9$, from those obtained by Eq. 17.

The K-values of Eq. 6 are characteristic of deformation when the algebraic sum of the areas of the two segments below and above the initial condition of the cable equals zero, or when the deformation occurs without change in H, without stretch of cable, without movements of the towers, and without involving the side spans. For these reasons, Eqs. 16 and 22 cannot reliably be used for estimating live-load deflections and rotations which, besides being influenced by the enumerated changes, are also affected by the ratio of live load to dead load. The close agreement in Example 2 and Example 4 between the results obtained from these equations and those obtained from the deflection theory is accidental as shown in Table 12.

NOTE.—This paper by D. B. Steinman was published in November, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1944, by Edward Adams Richardson; March, 1944, by R. K. Bernhard; May, 1944, by J. M. Robertson, and C. H. Gronquist; June, 1944, by F. B. Farquharson; and September, 1944, by H. Reissner, Blair Birdsall, Hardy Cross, and Frederick C. Smith, Hermann Friedlaender, and George S. Vincent.

[74] Cons. Engr., New York, N. Y.

[74a] Received by the Secretary August 31, 1944.

[75] "Handbuch der Ingenieurwissenschaften," by Joseph Melan, Vol. 5, 3d Ed., Leipzig, 1906, pp. 208–214.

The application of Eqs. 16 and 22 to a number of bridges in addition to those in Table 12 resulted in deviations, up to a maximum of 48%, from the values obtained by the deflection theory. These deviations also prove that the author's physical interpretation of the $\frac{2K}{b}$-values (as the loading from tower to midspan in pounds per square foot which produces a 1-ft quarter-point

TABLE 12.—APPLICATION OF EQS. 16 AND 22 TO VARIOUS BRIDGES

Bridge	(*a*) Eq. 16 Compared with Deflection Theory					(*b*) Eq. 22 Compared with Deflection Theory[a]			
	K (Eq. 6)	p (lb per ft)	$\eta_{0.25}$ by: Eq. 16 (ft)	$\eta_{0.25}$ by: Deflection (ft)	Percentage increase	K_1 (Eq. 21*a*)	θ by: Eq. 22 (radians)	θ by: Deflection (radians)	Percentage increase
Golden Gate	116	100	0.86	0.68	+27	470,000	0.0213	0.0168	+27
	126	2,000	15.87	12.41	+28				
George Washington	244	100	0.41	0.31	+33	1,370,000	0.0073	0.0055	+33
	259	2,000	7.42	5.88	+31				
Bronx-Whitestone	141	100	0.71	0.61	+17	388,000	0.0258	0.0223	+16
	156	1,500	9.62	8.41	+15				
Transbay	324	100	0.31	0.40	−22	708,000	0.0141	0.0184	−23
	350	3,000	8.56	10.70	−20				

[a] In Eq. 22, T = 10,000 lb-ft per ft.

deflection of stiffened suspension bridges) is incorrect. The percentages given are indicative of the accuracy of this interpretation.

The basic cable term of Eq. 6, $\frac{l^2}{H} = \frac{8f}{w}$, however, can be used for the calculation of quarter-point deflections of the unstiffened cable, due to the loading mentioned, if it is considered to be inextensible and fixed at the towers. For this condition, $\eta_{0.25} = f\left(4 + \frac{8w}{p}\right)^{-1}$. For $p = 100$, considering the prevailing dead loads, $\frac{w}{p}$ is between 12 and 150 and, even in the extreme case, the average value, $\eta_{0.25} = 1.525\frac{l^2}{H}$, is a close approximation ($\pm 2\%$) of the correct values.

To connect the stiffness and aerodynamic behavior of suspension bridges, the author evaluates Eq. 6 for twenty bridges, divides these values by the width, and lists the bridges in order of the magnitude of these values (Table 2). From experience the author states that stability requires these K per foot values to be not less than 15 for bridges stiffened by plate girders and not less than 6 for truss-stiffened bridges. In this manner, an empirical connection is established between the $\frac{2K}{b}$-values and the aerodynamic oscillations of suspension bridges.

The use of the basic terms of Eq. 6 to express the stiffness of a suspension bridge cable and its stiffening girder has been practicable in suspension bridge design. The concept of extending these terms by the corresponding first harmonic coefficients and dividing their sum by the width of the bridge is new; as is the significance ascribed to the resulting values by the author.

The practical meaning of the author's requirements for the minimum values of $\frac{2K}{b}$, for the $\frac{l}{d}$-ratio of 100, and for the minimum values for K and R can best be discussed by the application of these rules to several suspension bridges representing the span range of structures actually built.

Table 13 demonstrates that, for spans longer than 1,000 ft, the requirements

TABLE 13.—APPLICATION OF THE AUTHOR'S MINIMUM REQUIREMENTS FOR $\frac{2K}{b}$-VALUES AND $\frac{l}{d}$-VALUES TO VARIOUS BRIDGES

Bridge	Type	Center span (ft)	(a) Girder as Built			(b) Minimum Requirements				
			Depth (ft)	I (in.^{2}-ft^2)	K (Eq. 6)	Depth (ft)	I (in.^{2}-ft^2)	K (Eq. 6)	$\frac{d}{b}$	Increase in cable area (%)
Golden Gate.....	Truss	4,200	25.0	44,000	116	42.0	520,000	270	0.47	75
Tacoma Narrows.	Plate girder	2,800	8.0	1,000	62	28.0	165,000	300	0.72	157
	Truss	2,800	....	....	..		50,000	120		33
Bronx-Whitestone	Plate girder	2,300	11.0	2,615	140	23.0	156,000	550	0.32	117
	Truss	2,300	....	....	..		38,000	222		18
Deer Isle.....	Plate girder	1,080	6.5	424	69	10.8	6,000	300	0.46	75
	Truss	1,080	....	....	..		1,860	120		4
Thousand Islands.	Plate girder	800	6.0	252	127	8.0	1,600	300	0.26	21
	Truss	800	....	....	..		330	145		0

result in plate girders of impractical dimensions. For the Golden Gate Bridge, trusses would be required which would nearly double the cost of the structure. This bridge has been exposed for seven years to the most unfavorable wind conditions experienced by any of the large suspension bridges and has exhibited dynamic stiffness sufficient for its use and safety. Observed oscillations for winds of from 60 miles per hr to 78 miles per hr were quickly damped out when the wind velocity decreased. Therefore, the Golden Gate Bridge may be assumed to represent the limit of satisfactory dynamic behavior, contradicting the general validity of the author's dynamic stability criteria.

The requirement that plate-girder stiffened bridges be 150% stiffer than truss-stiffened bridges eliminates their use. The requirement that $\frac{l}{d} = 100$ has the same effect because it is destructive to advantageous $\frac{d}{b}$-ratios. Nor could plate girders be used for spans longer than 1,000 ft if $\frac{l}{d} = 100$ and an H-section had to be maintained. Nevertheless, the author concentrates on the

investigation of this section which, in view of experience and his recommendations, has lost its practical importance.

The author's analysis of the aerodynamic excitation of vertical and torsional oscillations is based on the characteristic drag, lift, and torque curves obtained on section models of the Tacoma Narrows Bridge. By idealizing the laboratory results, the author obtains a family of lift graphs corrected for drag (Fig. 5). The introduction of the K-values into the frequency, as well as into the energy, relations of vertical and torsional oscillations has the effect of establishing the connection between s' (the slope of the corrected lift graph) or s_t (the slope of the torque graph)—both for $\alpha = 0$—K, V, β, and the aerodynamic amplification ratio δ. The arbitrary function, β, of the phase difference across the section is also introduced for vertical oscillations to explain the behavior of the Tacoma Narrows Bridge in the low-velocity range. The values of δ_n, derived from the moment of inertia of the girders equated with δ, result in Eqs. 44 to 46 for the critical velocities of vertical oscillations and in Eqs. 62 to 64 for the critical velocities of torsional oscillations.

The author proves the validity of these equations by examples which give values corresponding to the observations on the Tacoma Narrows Bridge. Applying the equations to other bridges, however, the results are not convincing. For the Thousand Islands, Bronx-Whitestone, and Golden Gate bridges $\left(\frac{0.5\,d}{b} = 0.139\right)$, Eq. 45*b* with $n = 9$ yields 48, 85, and 70 miles per hr, and Eq. 46*b* with $n = 2$ yields 5, 1.7, and 2 miles per hr, respectively. The Tacoma Narrows Bridge subjected to a wind velocity of 42 miles per hr changed from a vertical oscillation with $n = 9$ to a violent torsional oscillation with $n = 2$. The maximum critical velocities determined by the author's equations were experienced by the enumerated bridges without any such effects. For this reason, the practical meaning of Eqs. 45 and 46 is not apparent.

Eqs. 62, 63, and 64, which show increasing safety from torsional oscillations, with decreasing width, also do not give results consistent with the behavior of actual structures. For more than five years the Bronx-Whitestone Bridge was exposed to wind velocities as great as 95 miles per hr, but at no wind velocity did torsional oscillations occur. Using Eq. 62*a*, the critical velocity of torsional oscillations of this bridge is 18.2 miles per hr; and, using Eq. 64*a*, 18.5 miles per hr. Assuming that everything is unchanged except that the bridge is narrowed to half of its width, then, for $\frac{b^2}{w} = 0.25$, Eq. 62*a* results in a critical velocity of 20.8 miles per hr, and Eq. 64*a* yields a critical velocity of 21.6 miles per hr. Such a drastic variation in the $\frac{b^2}{w}$-value did not change the critical velocities appreciably.

The evaluation of observations on the Tacoma Narrows Bridge and on its section models failed to give generally valid relations even for the solid H-section. The author's analysis is more interesting from the point of view of procedure than of practical applicability to design. This fact is apparent from "Part 4.—Methods of Securing Rigidity and Aerodynamic Stability," in which the proposed measures do not follow from the application of the results of the presented

analysis. The condition $\delta - \delta_s = 0$ is a basic relation in the critical velocity equations (see heading, "Part 2.—Aerodynamic Excitation of Vertical Oscillations: Structural Damping") where δ_s is essentially a function of R, the proportion of K due to I (the moment of inertia of the girder sections). In building up the resistance of the structure to aerodynamic action with stay ropes, the author assigns a new meaning to δ_s by explaining (see heading, "Part 4.—Methods of Securing Rigidity and Aerodynamic Stability: Use of Longitudinal Stays") the mechanical interference of stays with oscillations; and then writes (but without derivation) Eq. 65 for the amount that K is augmented by the stays. Eq. 65 is similar to the cable term, and not to the girder term, of Eq. 6, S being a function of a unit stress and the initial stay rope sag. Hence, the addition of this quantity to the girder term in the equation for K, to show a desired degree of increase in R is not warranted by the author's explanations or by the fact that pre-stressed stay ropes stiffen a suspension bridge to a certain extent. With the recommended minimum $K = 300$, the cost of the Tacoma Narrows Bridge, stiffened by plate girders, would be more than doubled (see Table 13); whereas by application of 16 sq in. of wire rope strand, in the form of longitudinal diagonal stays, the K-value of the bridge that failed would be raised to 332 and R would be increased to 0.816. In other words, a structural damping well above that of the Williamsburg Bridge would be secured at insignificant expense. This result, which is of great economic importance, demands a more thorough proof than the explanations in the paper. Experience with stays and reference to the Brooklyn Bridge, which was stiffened by six trusses, without stays, having values of $K = 190$ and $R = 0.22$, do not prove the author's claims.

The author's reference to the interference of transverse diagonal stays and top struts, with the out-of-phase movement of the cables, does not prove that Eq. 66*a* represents quantitatively the dynamical effectiveness of this arrangement (Fig. 9). The makeshift appearance of this detail (which has been used on ore-transport suspension bridges) does not befit the permanent and monumental character of a vehicular bridge structure.

The author's statement that the aerodynamic forces causing the vertical and torsional oscillations of the H-section can be eliminated or reduced to negligible proportions by leaving open spaces near the girders or by open-floor construction (see heading, "Part 4.—Methods of Securing Rigidity and Aerodynamic Stability: Use of Open-Floor Construction") is contradicted by experience and wind-tunnel tests. The Bronx-Whitestone Bridge was constructed in the following stages:[76]

(1) All steel, including reinforcing grid with floor plates attached, was erected and sidewalk areas were open, leaving two lateral vents with a total width of 13 ft $\sim \left(\frac{3\,b}{16}\right)$. At this stage, $\frac{b^2}{w} = 1.41$.

(2) Roadway concrete was poured and sidewalk areas were still open. At this stage, $\frac{b^2}{w} = 1.10$.

[76] *Construction Methods*, Vol. 21, June, 1939, p. 46.

(3) Sidewalk concrete was poured and the vents in stage (1) were closed. At this stage, $\frac{b^2}{w} = 1.0$.

In none of these stages, nor during the following five years, did the bridge have any torsional oscillations. Its largest vertical oscillations occurred, however, during stage (1) when, according to Eq. 60, the aerodynamic lift was reduced 60%, and, in addition, the multiplier β was reduced 40%.

Homer M. Hadley, Assoc. M. Am. Soc. C. E., describing experiments with the 100-ft scale models of the Tacoma Narrows Bridge, states:[77]

> "The steelwork of the original deck was reproduced to scale—8 ft. marginal stiffening girders, floor beams, stringers and wind trusses—but the floor slab and sidewalks were omitted. This model lacked 20 percent of the stabilizing weight of the prototype $\left[\frac{b^2}{w} = 0.67\right]$ * * *. It was tested with and without midspan diagonal stays. Without stays * * * it quite closely reproduced the vertical undulations of the prototype and torsional movements as well."

Neither closing nor opening the horizontal member of the solid H-section altered the aerodynamic behavior of the section as Eqs. 43 and 60 indicate; and the dimensional ratio $\frac{b^2}{w}$ cannot be offered as a measure of sensitiveness to torsional aerodynamic instability.

The structural arrangements, recommended by the author as methods for securing aerodynamic stability, have been used on actual bridges and in proposed designs for many years. It is not proper to call the H-section "conventional." Many existing suspension bridges have shallow, deck plate-girder, suspended structures, and it is purely accidental that long-span bridges using this form have not been built. Such suspended structures were proposed for the 3,000-ft Wilmington Bridge, in Delaware, and also for the 4,700-ft Battery-Brooklyn Bridge in New York, N. Y., in 1939, because this design satisfied the requirements of those bridges better than the H-section. All stay systems (center, longitudinal, and transverse, Figs. 8 and 9) discussed by the author were suggested by others, or were tested in practice long ago, and then as now they have proved themselves mostly inefficient. In 1926, open sidewalk spaces were used in the design of the proposed bridge over the First Narrows, Vancouver, B. C., Canada, by Henry C. Tammen, M. Am. Soc. C. E. Also, the 1,050-ft Otto Beit Bridge, built in 1938, over the Zambezi River in Rhodesia, Africa, is provided with lateral vents of width $\frac{3\,b}{16} \sim 6$ ft. Outside sidewalks have been used on bridges too numerous to list. Static or structural considerations suggested all these arrangements. Their effectiveness for securing aerodynamic stability is not supported by experience or experimental evidence.

Simple relations derived from the rigidity characteristics of a basic structure actually constructed seem to gage the sufficiency of stiffening trusses for

[77] *Engineering News-Record*, Vol. 131, 1943, p. 664.

aerodynamic stability. To establish such empirical relations, the dimensional properties of the bridges will be expressed by the cable and girder stiffness parameters: $\pi_1 = \frac{l^2}{H} = \frac{8f}{w}$ and $\pi_2 = \frac{l^4}{E\,I}$, and, by their ratios, the dimensionless parameters: $\pi_3 = \frac{\pi_2}{\pi_1} = \frac{H\,l^2}{E\,I}$ and $\pi_4 = \frac{\pi_2}{\pi_{1B}}$. The parameter π_{1B} refers to the basic structure. The writer utilized these parameters by equating the π_3-values of the Golden Gate and Bronx-Whitestone bridges, thus deriving the value of I required to stiffen the latter structure in 1941. At a conference on November 3, 1941, Shortridge Hardesty, M. Am. Soc. C. E., who was asked to pass on the sufficiency of the design, outlined a dependable deflection method on which he based his recommendations. For two basic structures, Golden Gate and Lions Gate, the two methods gave identical results. General utilization of these parameters will be effected by means of a representation in which the π_3 and π_4 values of the bridges are referred to parabolic curves obtained by assuming the span length to be variable in the following expression of the π_3-value of the basic structure:

$$\pi_{3x} = \pi_{4x} = \left(\frac{H}{E\,I}\right)_B (l_x)^2 \ldots\ldots\ldots\ldots\ldots\ldots (122)$$

Eq. 122 contains the aerodynamic and structural damping characteristics of the basic structure and indicates, for another structure of similar aerodynamic characteristics, the corresponding amount of structural damping that will assure stability, in the form of a requirement for the moment of inertia of the girders. For the reasons stated in connection with Table 13, the Golden Gate Bridge was selected as the basic structure. Introducing its $\frac{H}{E\,I}$-value into Eq. 122, a criterion of satisfactory dynamic behavior is obtained as follows:

$$\pi_{3x} \leqq \pi_{4x} \leqq 0.3872 \times 10^{-4}\,(l_x)^2 \ldots\ldots\ldots\ldots\ldots\ldots (123)$$

and the required moment of inertia of the stiffening trusses becomes:

By considering π_{3x} and solving Eq. 123 for I_x—

$$I_x = \left(\frac{I}{H}\right)_B H_x = 0.00089\,H_x \text{ (in.}^2\text{-ft}^2\text{)} \sim 0.0009\,H_x \text{ (in.}^2\text{-ft}^2\text{)} \ldots (124a)$$

by considering π_{4x} and solving Eq. 123 for I_x—

$$I_x = \left(\frac{I}{l^2}\right)_B (l_x)^2 = 0.00499\,(l_x)^2 \text{ (in.}^2\text{-ft}^2\text{)} \sim 0.005\,(l_x)^2 \text{ (in.}^2\text{-ft}^2\text{)} \ldots (124b)$$

Table 14 contains the numerical values of parameters π_1, π_2, π_3, and π_4 for twenty-four suspension bridges.

The π_1-values are plotted in Fig. 22. A circle is placed around the plots of the bridges that moved. Fig. 22 indicates that oscillations occurred in the full range of cable stiffness variation of bridges actually built. It shows that cable

TABLE 14.—VALUES OF PARAMETERS π_1, π_2, π_3, AND π_4

No.	Bridge[a]	Center span l (ft)	$\pi_1 \left(\frac{ft^2}{lb}\right)$	$\pi_2 \left(\frac{ft^2}{lb}\right)$	π_3 (non-dimensional)	π_4 (non-dimensional)
1	Davenport	740	0.1349	2.817	20.9	15.8
2	Maumee River[a]	785	0.0598	2.454	41.1	13.7
3	Waldo-Hancock	800	0.1662	8.357	50.0	46.8
4	Thousand Islands[a]	820	0.2000	28.024	140.1	157.0
5	Lempa River[b]	949	0.2018	4.771	23.6	26.7
6	Grand Mère	949	0.3088	6.279	20.3	35.2
7	Isle d'Orleans	1,059	0.2703	5.163	19.1	28.9
8	Deer Isle[a]	1,080	0.3600	56.659	157.4	317.4
9	Mount Hope	1,200	0.1811	8.392	46.3	47.0
10	St. Johns	1,207	0.1207	7.773	64.4	43.5
11	Triborough	1,380	0.0576	4.169	72.3	23.4
12	Manhattan	1,470	0.0515	0.917	18.0	5.1
13	Mid-Hudson	1,500	0.1364	5.819	42.7	32.6
14	Lions Gate	1,550	0.2609	21.174	81.2	118.6
15	Williamsburg	1,600	0.0829	0.924	11.1	5.2
16	Philadelphia	1,750	0.0615	1.916	31.1	10.7
17	Ambassador	1,850	0.1347	12.622	93.7	70.7
	Bronx-Whitestone:					
18	Present[c]	2,300	0.1455	181.795	1,249.5	1,018.4
19	Proposed[c]	2,300	0.1344	24.324	181.1	136.3
20	Transbay	2,310	0.0975	6.298	64.6	35.3
	Tacoma Narrows:					
21	1940[c]	2,800	0.3262	1,044.090	3,200.8	5,848.6
22	Proposed[c]	2,800	0.1600	19.500	121.9	109.2
23	George Washington	3,500	0.0872	....	....	....
24	Golden Gate	4,200	0.1786	121.932	683.0	683.0

[a] All bridges are truss type except those numbered 2, 4, 8, 18, and 21 which are plate-girder types. Item 19 combines both types. [b] Estimated from insufficient data. [c] Data supplied by Glenn B. Woodruff, M. Am. Soc. C. E.

stiffness does not govern dynamic stability and demonstrates that weight is not only the most expensive but also is ineffective in controlling aerodynamic oscillations.

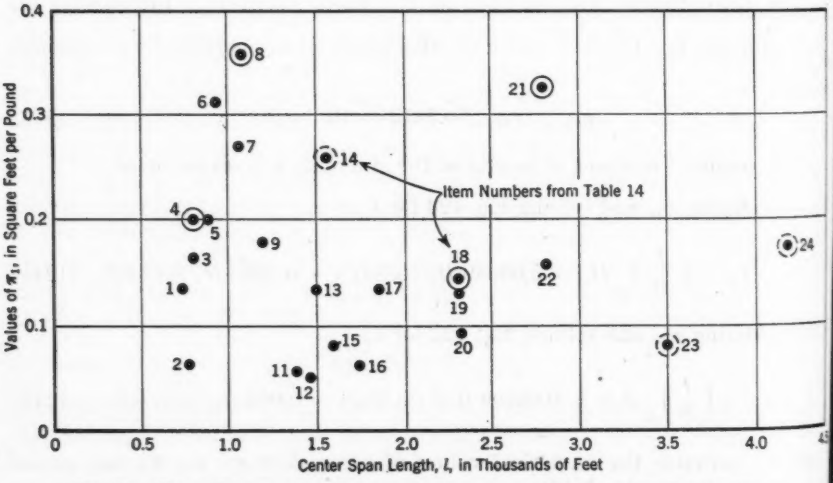


FIG. 22.—CABLE STIFFNESS OF SUSPENSION BRIDGES, AS MEASURED BY PARAMETER π_1 (Full Circle Denotes Objectionable, Dotted Circle Denotes Not Objectionable Movements)

Eq. 123 is represented by a curve passing through the $(\pi_3 = \pi_4)$-value of the Golden Gate Bridge in Fig. 23. The plotted points in Fig. 23(*a*) correspond

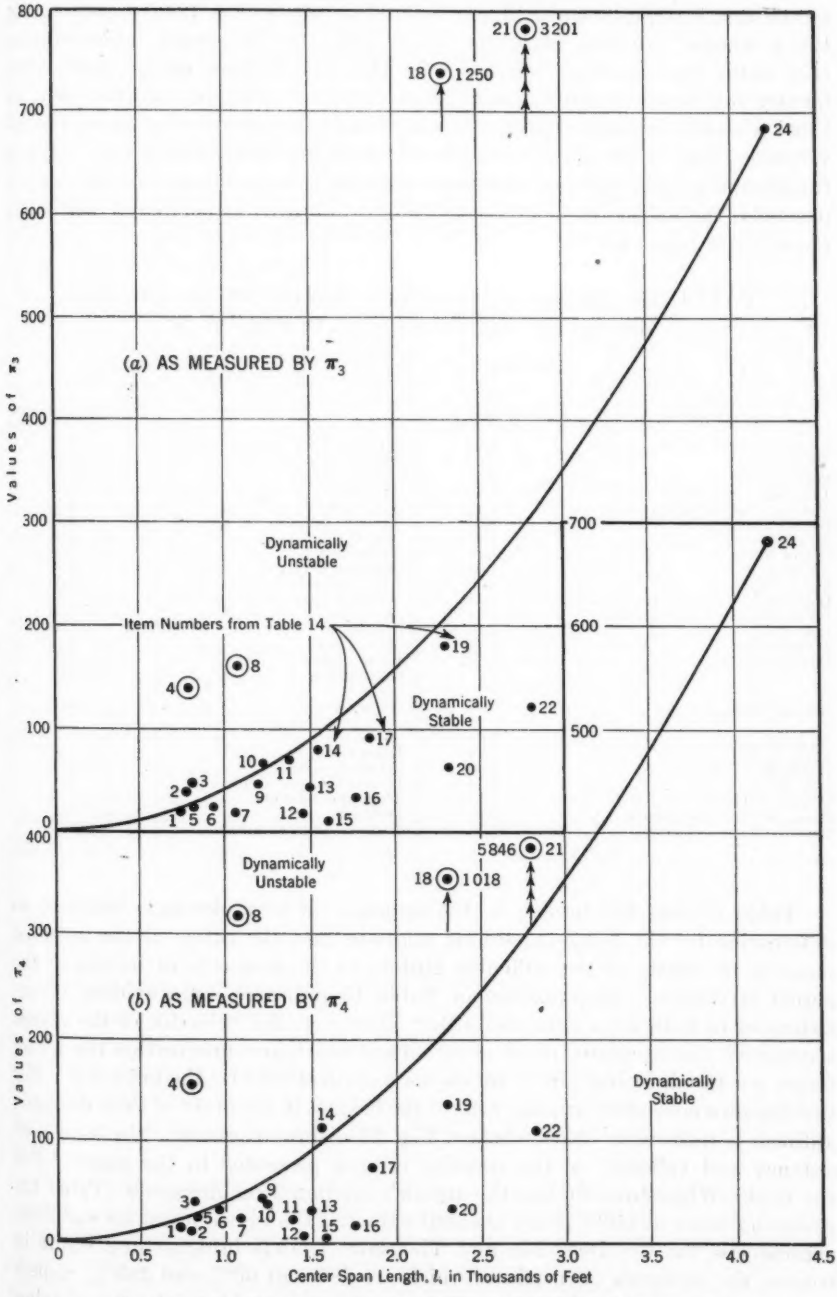


FIG. 23.—DYNAMIC STIFFNESS OF SUSPENSION BRIDGES, AS MEASURED BY DIMENSIONLESS PARAMETERS π_3 AND π_4

to the first interpretation of the criterional parabola (Eq. 124*a*), representing the π_3-values. In Fig. 23(*b*) they correspond to the second interpretation (Eq. 124*b*), representing the π_4-values. The plots of those bridges with satisfactory resistance to aerodynamic action are near or below, and the plots of those with objectionable movement are considerably above, the curve, the divergence being nearly proportional to the character of the oscillations. When the distances of the plots are measured with the corresponding ordinates of the parabola, the deficiency or excess stiffness is indicated as compared with that required by Eqs. 124.

TABLE 15.—RATIOS OF REQUIRED MOMENT OF INERTIA AND MOMENT OF INERTIA IN ACTUAL STRUCTURE

(Excess = +; Deficiency = −)

Bridge	Eqs. 123 and 124*a*	Bridge	Eqs. 123 and 124*b*
Williamsburg	+ 8.93	Williamsburg	+ 19.15
Manhattan	+ 4.65	Manhattan	+ 16.29
Philadelphia	+ 3.81	Philadelphia	+ 11.05
Transbay	+ 3.20	Transbay	+ 5.85
Tacoma Narrows (proposed)	+ 2.49	Triborough	+ 3.16
Isle d'Orleans	+ 2.27	Tacoma Narrows (proposed)	+ 2.77
Mid-Hudson	+ 2.04	Mid-Hudson	+ 2.67
Grand Mère	+ 1.73	Ambassador	+ 1.87
Ambassador	+ 1.41	Maumee River	+ 1.77
Mount Hope	+ 1.20	Isle d'Orleans	+ 1.50
Lions Gate	+ 1.14	Bronx-Whitestone (proposed)	+ 1.50
Bronx-Whitestone (proposed)	+ 1.13	Davenport	+ 1.34
Lempa River	+ 1.10	St. Johns	+ 1.30
Triborough	+ 1.01	Mount Hope	+ 1.19
Davenport	+ 1.01	Lempa River	+ 1.03
		Grand Mère	+ 1.01
Golden Gate	1.00	Golden Gate	1.00
St. Johns	− 1.14		
Maumee River	− 1.40	Lions Gate	− 1.28
Waldo-Hancock	− 2.02	Waldo-Hancock	− 1.88
Deer Isle	− 3.49	Bronx-Whitestone (present)	− 4.97
Thousand Islands	− 5.65	Thousand Islands	− 6.33
Bronx-Whitestone (present)	− 6.10	Deer Isle	− 7.03
Tacoma Narrows (1940)	− 10.54	Tacoma Narrows (1940)	− 19.27

Table 15 lists the bridges in the sequence of their dynamic stiffness, as determined by the foregoing simple relations and the ratios of the required moment of inertia of the stiffening girders to the moments of inertia in the actual structures. Examination of Table 15 indicates some evident inconsistencies in both Eqs. 124*a* and 124*b*. However, the behavior of the actual structure is the integrated result of action and resistance since neither the acting forces nor the damping effects are shown quantitatively by the behavior. For this reason a consistent arrangement of the bridges in the order of their dynamic stiffness is unknown. Nevertheless, Fig. 23 disproves visibly "the high consistency and validity" of the stability criteria presented in the paper. For the Bronx-Whitestone Bridge, the author's minimum requirements (Table 13) result in trusses of 140% larger moment of inertia than is sufficient for stability; contrariwise for the Deer Isle and Thousand Islands bridges they result in trusses, the moments of inertia of which are deficient 60% and 380%, respectively, as indicated by Fig. 23. (Bridges that are stable without satisfying

this criterion have been discussed by the writer elsewhere.[78]) Any attempt, however, to derive aerodynamic stability from purely empirical considerations should be regarded as a temporary expedient for estimating purposes and should not take the place of specific experimental research for each structure to be designed.

Because the weight of the various characteristics affecting economical design for aerodynamic stability is not apparent from the results reported in the paper, the importance of the variation of the characteristics of the suspension bridge system as they influence aerodynamic stability will be examined. This examination reveals the following:

(1) Cable sag ($\sqrt{f}$) determines the fundamental frequency; hence the actual frequency variation is very small and its effect on stability irrelevant.

(2) Cable stiffness, predominately, determines the structural elastic stiffness. Consequently the requirements of static stiffness can often be met fully without trusses. Fig. 22 shows that within the limits of its practical variation, cable stiffness cannot assure aerodynamic stability.

(3) Trusses, within economically practical limits, contribute little to the structural elastic stiffness but they can be the principal contributors to structural damping. They represent, however, neither a progressive, nor always an economical, solution in the design for aerodynamic stability.

(4) Stay systems may be developed into efficient contributors to structural damping. Dynamic models overestimate the damping efficiency of stay systems grossly. The Bronx-Whitestone model indicated high damping effect for stays which, installed on the prototype, have been found insignificant. Wind-tunnel models equipped with stays should supply basic data for the development of efficient stay systems. Stay systems will then gain importance in the economical design for aerodynamic stability.

(5) Suspended structures which are aerodynamically stable, with or without efficiently provided additional structural damping, constitute the economical solution of the design problem of aerodynamic stability. In other words, the structure whose satisfactory static rigidity has been achieved the most economically will also be aerodynamically stable, or can be made so the most economically.

The author's equations are useless for the solution of the practical design problem, because neither the variation of aerodynamic damping with arbitrary modification of the shape of the suspended structure, nor the variation of structural damping resulting from any practical arrangement, can be calculated by them. In fact, they give inconsistent results even for shapes similar to the one from which they were derived, as proved by the behavior of existing bridges.

The author's conclusions recommending that stability be secured by increasing weight and depth must be rejected. The fundamental design principle of structural efficiency, expressed by lowering the ratio of the weight of metal in the structure to the live load the structure is able to carry, should never

[78] *Engineering News-Record*, Vol. 131, 1943, pp. 876–877.

be abandoned, especially because weight, within any reasonable limit, does not assure dynamic stability. Analysis of the cost items of long-span suspension bridges clearly indicates the great economic significance of the reduction of weight. Deep girders also affect the cost unfavorably. The determination of stability, therefore, is primarily the task of aerodynamic research which should be connected directly with the design of suspension bridges until the experiments permit a clear theoretical interpretation.

Corrections for *Transactions:* In November, 1943, *Proceedings*, on page 1384, lines 12 and 15 following Eq. 56*b*, change "aerodynamic" to "significant" and "minimum" to "critical," respectively.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

STORAGE AND THE UNIT HYDROGRAPH

Discussion

BY DON JOHNSTONE

DON JOHNSTONE,[35] ASSOC. M. AM. SOC. C. E.[35a]—The difficulties of winnowing out the grains of wheat in technical literature are certainly not decreased by inventing new meanings for old phrases. Engineering terms should mean the same thing today as yesterday. If new concepts are introduced, new terms should be invented. The author, in applying the term "unit hydrograph" or "unitgraph" to a hydrograph which includes ground-water flow, is misusing a basic definition. Similarly, in applying the term "concentration time" to the time interval "between cessation of runoff-producing rainfall and the minimum indicated value of K" he is tying another time-honored phrase to a new concept. Mr. Bernard's definition of concentration time ("a time interval at the end of which all parts of the watershed are contributing to the flow at the point of observation") gives that particular combination of words a job to do and should pre-empt it from other uses.[36] Other examples of criticizable terminology include the use of "runoff" for "rainfall" in the third and fifth paragraphs of the "Introduction" and in the third paragraph of the section headed "Valley Storage and the Unit Hydrograph," followed by its use in an entirely different sense in the second paragraph of the next section; and the use of "the channel" in that same section without indicating whether reference is to the main stream alone or to any "channel" in the watershed.

Passing to other matters, attention is invited to the statement in the "Introduction" that the determination of unitgraphs from "multiple periods of runoff [sic] * * * is indeterminate, as there are always more equations than unknowns and a multiple of possible solutions." Presumably the author meant "more unknowns than equations," but, waiving that point, the statement is still incorrect. By unitgraph theory, surface runoff in the xth period

NOTE.—This paper by C. O. Clark was published in November, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1944, by James S. Sweet, and Otto H. Meyer; March, 1944, by L. K. Sherman, Gordon R. Williams, and Ray K. Linsley, Jr.; May, 1944, by E. F. Brater, L. C. Crawford, Robert E. Kennedy, and Victor H. Cochrane; June, 1944, by Franklin F. Snyder; and September, 1944, by Howard M. Turner.

[35] Asst. Prof., Civ. Eng., Ohio State Univ., Columbus, Ohio (on military leave).

[35a] Received by the Secretary November 1, 1944.

[36] "An Approach to Determinate Steam Flow," by Merrill Bernard, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 350.

resulting from a storm of n periods duration is $c_1 f(x) + c_2 f(x-1) + \cdots + c_n f(x-n+1)$, in which $c_1, c_2, \cdots, c_n$ are the net rainfalls for the corresponding periods and $f(x), f(x-1), \cdots, f(x-n+1)$ are the ordinates of the unitgraph for the xth, $(x-1)$th, $\cdots$, $(x-n+1)$th periods. If flow measurements and rainfall measurements were precise, if the base flow (ground-water flow) were known, if the net rainfalls for all periods were determinable, and, finally, if the stream behaved exactly in accordance with unitgraph theory, it would be possible to derive the unitgraph from the hydrograph of a multiple-period storm by successive subtractions. Actually, successive subtractions yield nothing of value, as none of the four "if's" is exactly in accordance with fact and as the process involves the accumulation of error from step to step. The concept is set forth here, however, to demonstrate that there are actually as many equations available as there are unknowns (so far as unitgraph theory is concerned), and that there should therefore be no possibility of multiple solutions from a given set of data if a proper technique of derivation is applied. One such technique has been outlined by William T. Collins,[37] Jun. Am. Soc. C. E.

It follows (see "Introduction") that "the practical requirement of finding an isolated flood hydrograph resulting from a uniform unit rainfall," which the author states "provides an almost insurmountable obstacle to the derivation of the unitgraph for very small [?] drainage areas and very short unit periods [intervals] of time," is no longer a requirement at all. As a matter of fact, a protracted period of heavy rainfall is the ideal period from which to derive the unitgraph, because (*a*) rainfall is more likely to be nearly uniform over the watershed, (*b*) infiltration conditions are more nearly uniform throughout the period of rainfall, and (*c*) the fact that rainfall actually extended over the selected unit time interval is assured. To carry this argument further would be beyond the proper scope of discussion; the writer acknowledges that despite the incorrectness of the quoted statements there is still need for a method that will permit deriving unitgraphs without reference to rainfall records.

The argument under "Valley Storage," down to the introduction of Eq. 4, is confusing. Particularly welcome would be a justification of the statement that "reduction of the inflow rate necessarily would be accompanied by * * * an increase of outflow rate." Fig. 1 and its explanation are unconvincing, and the writer would like to see (here as elsewhere) proof in the form of actual recorded data rather than in the form of a rather casual theoretical analysis.

The argument on the extension of storage theory to ground-water flow appears to "boil down" to this: It would be convenient if unitgraph principles were applicable to ground-water flow; therefore they are applicable. Without further proof, the author proceeds to apply them. There is something about neatly arranged tables of computations that seems to consecrate the result, and it is likely that computing a number of ground-water flow hydrographs by this method would confirm many a budding hydrologist in the belief that the implied relationships had been proved. Actually, the "refinement" attained by treating ground-water flow in this manner (if it be, in fact, a

[37] "Runoff Distribution Graphs from Precipitation Occurring in More than One Time Unit," by William T. Collins, *Civil Engineering*, September, 1939, p. 559.

refinement at all) seems scarcely worth the effort; 30% of curve (c) in Fig. 8 could be replaced by the customary sloping straight line and the change in curve (d) would scarcely be noticeable.

An even more basic criticism, however, is the inclusion of ground-water flow, by whatever means computed, in the unitgraph. By including it, the range of usefulness of the unitgraph is seriously impaired. Its major function is not—as emphasis in many papers would lead one to believe—to confirm itself repeatedly by reproducing second-rate floods with more or less accuracy. Rather, it is a tool to indicate what will probably happen when a really first-rate rainfall comes along; and it should be left to do that job unhampered by the introduction of ground flow factors that will likely be inapplicable at the big moment. Suppose the maximum storm hits the watershed when the ground is saturated—or frozen. A true unitgraph can handle that situation (compare the "pluviagraph" of Mr. Bernard[38]); a composite graph of surface and subsurface runoff cannot handle it—and it errs on the side of danger.

Passing to the author's example, there is much difficulty in following the detail. Table 1 shows $K = 15$, whereas the text states that $K = 9$. Also, the relationship between the plotted curves of Fig. 8 and the data of Table 1 is obscure. Should not curve (d) be the plotting of Col. 9?

However, the major shortcoming of the example is the failure to explain the construction of Fig. 6. This is the heart of the entire matter—the drawing of lines on a map in such a manner as to define areas that make their contributions to "the channel" in successive intervals of time. How is it done? What factors are taken into account? Why do the area boundaries cross the streams without bending stream-upward, when logic suggests that they should take the form of exaggerated contour lines? (See, for example, Mr. Bernard's statement:[36] "* * * the efficiency of the main channel, through higher velocities, places headwater areas much closer to the outlet in terms of time than extensive areas nearer but less advantageously located.") With a knowledge of how to chart these areas, the operator finds the remainder of the technique to be a matter of measuring areas by planimeter and of running a calculating machine; without it, he is at a loss where to start.

The justification for applying the same value of K to the contributions of all areas is not apparent. It would appear to be reasonable only on the assumption that storage conditions are the same in all areas and that no storage exists in the main channel; but obviously the author does not make any such assumption. Rather, he takes for the value of K the value existing at the moment when all surface runoff is presumed to have reached "the channel." It follows that K for the downstream areas should be smaller, and for the upstream areas larger, than the average.

Since delineating the areas on the map is subject to a wide range of personal choice, and since the use of a single value of K is open to question, the writer suggests that a somewhat different approach might yield equally satisfactory results with less effort. Using "concentration time" as defined by Mr. Bernard, the base length of any hydrograph of surface flow (including the unitgraph as

[38] "An Approach to Determinate Stream Flow," by Merrill Bernard, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 358.

defined by Mr. Sherman) is equal to the duration of runoff-producing rainfall plus the concentration time. This statement includes the instantaneous unitgraph, for which the base length is simply the concentration time itself. Concentration time, by the Bernard definition, can be determined from an inspection of hydrographs, with reasonable accuracy, as readily as concentration time by the Clark definition. Having determined it, divide the watershed map into the corresponding number of time-interval areas and determine the percentage area of each. The tabulation of these percentages, without any further operations, is in itself the instantaneous distribution graph of surface flow for the watershed.

It cannot be said that the effect of storage is overlooked in this proposed method, for it is reflected in the fact that concentration time (the Bernard definition) takes in the entire period of time required for a particle of water to travel from the hydraulically-most-remote point to the point of observation. Basically, the analysis seems as sound as that of the author's more complex procedure. It would be interesting to determine a unitgraph for the Appomattox River by this proposed method and test it, also, for the flood of April, 1937. Unfortunately, the necessary basic data are not available to the writer. Also, an adequate technique for dividing the watershed map presumably remains to be developed. This should be a fruitful field for further inquiry.

In making such a test, it would be necessary to assume a "reasonable" ground-water flow, and the writer would be content with a straight line joining the point on the observed hydrograph at which rainfall begins, with the point at which surface runoff ceases. Fig. 9 indicates the latter to be at the line marked "April 3."

Corrections to *Transactions:* See the errata published in *Proceedings* for February, 1944, p. 251, and March, 1944, p. 409.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

GEOLOGY IN HIGHWAY ENGINEERING

Discussion

BY HYDE FORBES, A. A. EREMIN, AND MARSHALL T. HUNTTING

HYDE FORBES,[23] M. AM. SOC. C. E.[23a]—It is a matter of satisfaction to the writer that the Society has published another paper in the interest of applied geology. The paper is specific in its application to highway engineering, but the treatment is broad enough to apply to many fields in which civil engineers are active. It is of distinct value in further calling the attention of engineers and educators to the application of geology in engineering development and construction.

The late J. C. Branner, head of the Department of Geology at Stanford University, Stanford University, Calif., until his death in 1922, was a scientist of broad vision, and his efforts and accomplishments in the practical application of geology in the fields of petroleum and mining engineering are attested through the accomplishments of his students after graduation. The curriculum he developed was very broad, in that abridged courses in civil, mechanical, chemical, and electrical engineering were required, with opportunities to the individual for specialization in mining, metallurgy, petrography, geochemistry, ore deposits, paleontology, and petroleum.

The writer was particularly fortunate in having Professor Branner select him to direct in work in a new field in which the professor was endeavoring to obtain interest, research, and study; namely, Engineering Geology. After being graduated in 1913, the writer was employed under Professor Branner and was subsequently recommended by him to other similar positions prior to World War I. During these engagements, primarily directed to the investigation of the geology and ground-water conditions of certain areas, it was found that employers had problems involving geology upon which the engineers solicited the writer's services and advice.

NOTE.—This paper by Marshall T. Huntting was published in December, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1944, by W. W. Crosby; March, 1944, by Carl B. Brown; April, 1944, by F. H. Kellogg, Berlen C. Moneymaker, E. F. Bean, A. T. Bleck, Lyman W. Wood, Philip Keene, and Jacob Feld; May, 1944, by H. E. Marshall, K. B. Woods, and D. J. Belcher; June, 1944, by R. Woodward Moore, George E. Ekblaw, and Allen S. Cary and Ade E. Jaskar; and September, 1944, by Karl Terzaghi, D. P. Krynine, Robert F. Legget, and Irving B. Crosby.

[23] Cons. Engr. and Geologist, San Francisco, Calif.

[23a] Received by the Secretary October 16, 1944.

The first of these positions, with the Spring Valley Water Company of San Francisco, Calif., afforded opportunities to do work in connection with the watertightness and stability of the abutments for the Calaveras Hydraulic Fill Dam then under construction, and the physical and geochemical characteristics of the materials used in the fill. Similar opportunities were afforded in connection with the foundation investigation for, and the construction of, the base of the Gibralter Dam by the City of Santa Barbara, Calif., in 1915, when the writer was investigating ground-water problems. Professor Branner was consulted and, with the advice of consulting civil and hydraulic engineers contacted during those employments, the writer attempted the preparation of a curriculum aiming at collegiate specialization in engineering geology.

After Professor Branner died in March, 1922, the attempt to institute a curriculum at Stanford University emphasizing the practical application of geology in the field of engineering development and construction was not successful. However, the writer has continued to expand the outline for applied engineering geology which he established in association with Professor Branner and in the collection of material as his experience has broadened. His files and library are indexed and cross-indexed under the headings of that outline, in all of which he has made practical and technical application of the science of geology and under which he has found much of value in the engineering literature. The writer presents that outline here in acknowledgment of his debt to Professor Branner and in the hope that it will be of service to engineering educators:

(1) Geology of Rock Formations.
  (*a*) Character and structure—jointing and fractures.
  (*b*) Weathering—effective depth and geochemical change.
  (*c*) Interstitial binder—primary and secondary cementation, vein filling, healing of fractures.
  (*d*) Strength, specific gravity, watertightness, stability, solubility.
(2) Geology of Unconsolidated Formations.
  (*a*) Source of material and method of deposition.
  (*b*) Erosion, transportation, and silting.
  (*c*) Geochemical change and physical change in transportation and deposition, solution, oxidation, carbonization, hydration, abrasion, and wear; further changes to be expected.
  (*d*) Structure, porosity, and permeability.
  (*e*) Water content—drainage and recharge.
(3) Engineering Seismology.
  (*a*) Recognition and location of faults.
  (*b*) Determination of activity of faults and intensities of earthquakes.
  (*c*) Affect of vibrations on engineering structures.
  (*d*) Location of highways, pipe lines, and structures in reference to active faults.
  (*e*) Earthquake resistant construction.
(4) Applied Ground-Water Hydrology.
  (*a*) Occurrence, movement, and head of ground water.
  (*b*) Chemical and mechanical properties of ground-water movement and their effect upon the containing media; affect of ground-water pressure.
  (*c*) Development of water supplies—through physical works such as wells, spring development, tunnels, and galleries.

(*d*) Safe yield and expected life of ground-water developments.
(*e*) Spreading of water for ground-water recharge.
(*f*) Drainage of saturated ground.
(5) Landslides and Earth Movements.
(*a*) Occurrence, investigation, and correction.
(6) Soil Technology.
(*a*) Occurrence, and classification of soils.
(*b*) Development of soils and geochemical changes, from origin in rock fragments or products through stages in development.
(*c*) Soil mechanics—the mechanical properties of soils as foundation material and for use in engineering structures.
(7) Natural Structural Materials and Methods Adapted in Construction.
(*a*) Earth-fill construction, compaction, stability, geochemical change.
(*b*) Hydraulic fill construction, effect of water and transportation.
(*c*) Rock-fill construction, durability, insolubility, crushing strength.
(*d*) Subgrade for roads, highways, and air ports.
(8) Foundations.
(*a*) Geology of dam sites, requirements of structures.
(*b*) Foundations for bridges.
(*c*) Foundations for highways.
(*d*) Foundations for buildings and other structures.
(9) Tunnels and Tunneling.
(*a*) Investigation, location, design of support required.

If the engineering student is given an opportunity to obtain a basic knowledge in the foregoing subjects, he will enter his professional career well equipped to understand natural conditions and the use of natural materials, and, what is more important, with sufficient knowledge to be aware of his own limitations and to know wherein the services of a specialist are required. This lack is responsible for a large number of failures of structures. The geology student, on the other hand, will have his eyes opened to the possibilities of a specialization that will bring much satisfaction and opportunities for service. In the past, the engineer has made the mistake of assuming that a man who has acquired a college degree in geology "knows all the answers." A geologist cannot see underground any better than the next man, but he can recognize certain conditions from surface indications and, with adequate experience, will learn how to gain subsurface knowledge and apply his basic knowledge correctly.

Under the head of "Problems," the author lists twelve items in which the geologist's services might be profitably engaged. He then discusses these from the standpoint of an experienced engineering geologist. The writer agrees with the first premise, but the testing of material goes beyond the province of the geologist in the opinion of the highway engineer, and it is true that the geologist requires additional specialized training and knowledge in applying his basic training and knowledge to problems (1), (2), and (3). The geologist must have more than a basic training and knowledge of hydrography and ground-water hydrology, too often lacking in the collegiate training, in order to be of value in connection with problems (4), (6), (8), and (10).

Training and experience can be acquired just as well, and in as short a time, on engineering development work and construction as in the field of applied geology, which has been most often associated with geologists by the engineering

profession—namely, mining engineering and petroleum engineering. It has always appeared to the writer that the failure to apply geological training and knowledge properly in highway engineering was the cause of considerable economic waste.

Beginning in 1928 and continuing well into the depression years, the oil companies of the West found that the surface geological structures were practically all determined, overproduction was causing difficulties in the industry, and geological staffs were drastically cut. The older geologists were absorbed in administrative and executive positions, but the younger men were dismissed. Many of the latter came to the writer to discuss opportunities for employment and the broadening of their activities. The advice given was that they should take the examination for junior engineer in the State Highway Department and then make a job for themselves as an opportunity arose to show that the application of their special knowledge was of value. Several men took the examination, got on the lists, but were not called after long periods of waiting.

It was felt that some preference in the matter of hiring should be given men having such qualifications, and a letter was written to the Chief Engineer of the State Highway Department setting forth this matter and submitting an outline covering the application of this special knowledge in highway engineering. A conference was arranged at the State Capitol at which the subject was received with enthusiasm by the division engineers in attendance but with a lack of enthusiasm by the executive head. The writer was given to understand that such a setup would require a centralized control, which highway engineers were trying to avoid, or the retention of a consultant for the direction of the field men, which was against their general policy. As far as is now known, no benefits were derived from this effort.

In 1928 the writer was appointed to a committee of the San Francisco (Calif.) Section of the Society to study the foundation and subsidence question over the built-up area of San Francisco. The writer presented the viewpoint of the geologist, and these discussions, resulting in a report published subsequently by the Section, opened up to the writer a field of application of geology and study—namely, foundations for buildings on unconsolidated materials.

The author discusses the problem of acting as a witness in litigation (see problem (12) under the heading "Problems"). Litigation affords the time and money for extensive and exhaustive research such as is never available in engineering development and construction. Court presentation of the engineer's findings is subjected to the keenest, critical analysis in cross-examination. The writer has sometimes observed expert witnesses attempt to solve mathematically the problems of the underground. When purely mathematical solutions are attempted and broken down under cross-examination, or matched by one factually determined, it soon becomes apparent that nature does not act in an empiric manner and that empirical formulas do not apply. One is greatly impressed with the degree of confusion and the ridiculous results achieved when values of reasonable variance from those selected by the proponent, as assumption or small-scale test result, are substituted in a formula.

A mathematical computation, involving factors evaluated through assumption, or determined by small-size sample test, is no better than the value of that assumption or test. Its result might have been better stated as the proponents' unsupported opinion in the first place.

A study of geology reveals that geochemical change breaks down grain size with comparative rapidity. Under one set of conditions geochemical change occurs in natural material to produce a certain product and under similar, but slightly varying conditions, the change undergone by the same material yields a different product. Moisture contents of materials, determined as percentages of their oven-dried weights, do not take into consideration any of the water adsorbed or water that remains in fixed chemical combination. There is no constant law as to the quantity of water that may be taken up in chemical combination by the natural materials. For the foregoing reasons it is difficult to accept mathematical calculations based on sample tests.

There is no such thing as an undisturbed sample. The act of sampling implies disturbance from its position of confinement and support, and a new relationship to air—its oxygen, carbon dioxide, and moisture. The degree of disturbance can be ascertained only through knowledge of the geologic history, the physical condition and ground-water condition of the wider area sampled. That does not mean that sampling and testing should be abandoned. Boring and sampling, and more exploration boring, are needed to give as complete knowledge of the subsurface conditions as possible; but the limitations of the results obtained must be realized and used, within those limitations, to denote action, reaction, trends, and ranges, as tools to work with in the exercise of judgment. The writer uses shear tests, compression tests, physical, chemical, mechanical, and moisture analyses in his work for that purpose; but, in bearing-capacity determinations in foundation investigations, he depends upon positive vertical cyclic loading of the undisturbed foundation material; and for settlement he depends upon the experience recorded upon the same or similar foundation conditions determined through complete boring exploration.

To summarize, engineering education has not provided comprehensive curriculums in applied geology but has gone "overboard" on "soil mechanics." Former students and graduates of the Stanford Geology Department have told the writer, during the past several years, that only two fields of activity appear to be open to them—teaching and the United States Geological Survey. More than fifty graduates have joined the Survey, and, after World War II, with the restriction of appropriations such as occurred after World War I, many will be seeking new employment. If they are forced into commercial lines, as geologists were during the early 1930's, much training, knowledge, experience, and ability will be lost to the profession.

The profession as a whole does not recognize and appreciate the importance of applied geology in connection with engineering development and construction and it has overemphasized soil mechanics far more than its importance warrants.

A. A. EREMIN,[24] ASSOC. M. AM. SOC. C. E.[24a]—By illustrative examples, Mr. Huntting has shown the importance of geological investigations in highway construction. In the past few decades considerable interest has been expressed in the value of geology. In various universities the introduction of courses in soil mechanics and geology has increased the curriculum requirements for engineering students.

Economic considerations fully justify geological study for highway structures. Various devices have been developed to insure safety in structures. Streams are regulated and protected by various types of embankments. Bridge seats are designed to follow pier displacement resulting from the movements of various geological formations. In tunnel construction a temporary lining is used to study the characteristic features of the soil penetrated by the tunnel.

Without these engineering devices, geological science is still unable to yield reliable results. Further progress in soil mechanics and geology may be obtained through the cooperative effort of engineers and geologists by interpreting construction failures. A tendency still prevails to overshadow or to hide the real causes of some important structural failures. It is true that learning through failures is an expensive method; but an investigation of the failure of a structure designed according to standard assumptions is still the most reliable and effective method of improving design and construction. Many types of accidents are repeated constantly because no one studies the causes sufficiently. The final design of many large tunnels is the result of careful investigation of partial failures that occurred during construction.

MARSHALL T. HUNTTING,[25] ESQ.[25a]—Every thinker or writer must inevitably be biased by his own particular training and experience. Hence, the writer may have overemphasized the place of geology in highway engineering, or he may have placed relatively too much emphasis on certain subjects. The discussions charging overemphasis or improperly placed emphasis are subject to the same analysis, of course. One man charges that soil mechanics or soil science has been neglected; others object to including the subject of soils in a geological discussion. As was pointed out by Professor Terzaghi, soil mechanics is not a substitute for engineering geology. The opposite is also true. The two should be complementary, with soil tests and geological investigations combined, to obtain best results.

The experience of at least five discussers shows that clay is not a major constituent of the soils where frost heaving is worst. The formation of ice layers causes heaving. Wherever the water table is higher than the frost line, heaving will almost certainly occur, and the more permeable the soil the more water can be furnished to freeze into ice layers. Therefore, even gravels may be heaving soils. In fact, the sand and silt soils where the water table is high will be the worst offenders for that reason, and clay is preferable because of its nearly impervious "insulating" effect. On the other hand, in areas where the

[24] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[24a] Received by the Secretary October 31, 1944.

[25] Private, U. S. Army, Student Company G, Student and Service Regiment, Ft. Belvoir, Va.; formerly Geologist, State Div. of Geology, Dept. of Conservation and Development, Pullman, Wash.

[25a] Received by the Secretary November 9, 1944.

water table is well below frost level and heaving is less common the water must rise by capillarity to reach the trouble zone. In this case, the writer contends that clay, mixed with other soil constituents, adds to the capillarity of the soil and makes a given soil a frost-heaving offender, whereas, under identical drainage and water-table conditions, the same soil without its clay content would not cause the water to rise high enough to allow heaving to occur. To prevent frost heaving, the examiner first must make sure the water table is below frost line. Then he must balance two costs against each other:

(1) The cost of further lowering the water table to below the depth from which water may rise by capillarity to the frost level; and
(2) The cost of breaking the capillary rise from the water table to frost level by filling with gravel or by some other means.

Where sands, silty sands, and silts without clay are soils that heave, the writer suggests that this fact in itself may be sufficient evidence of a high water table and thus may simplify preventive studies by indicating directly the need for better drainage—no other preventive measure being applicable. Such reasoning would not necessarily apply in the case of silts, which have considerable capillarity even when clay is present in very small percentages.

Admittedly, the writer's statement (see heading, "Frost-Heave Problems") —"It has been shown that in many, if not most, instances of frost-heave disturbances clay is a major constituent of the subgrade material"—is in error. It should be amended by the addition of the clause "where the water table is well below the frost line." Also "major constituent" is much too vague a term for use when not accompanied by statistical data. Possibly, a fruitful line of research would be a study of the relation between soil particle size and capillarity, using samples taken from heaving soils. A comparison of the effects of the several individual classes of clay minerals would be important in such a study. A soil with a certain percentage of clay of one mineral composition might be expected to give results different from those given by soils with the same percentage of clays composed of other minerals.

How the soundness of a bridge superstructure can be independent of foundation materials, as stated by Mr. Feld, is difficult to conceive. A number of bridges have failed because piers and superstructure were not properly designed for the existing foundation conditions. In the discussion of bridge foundations, the writer did not wish to "indicate that taking and testing soil samples is entirely the work of geologists." His statement (see heading, "Competency of Materials for Bridge Foundations") was,

> "The geologist's sphere of usefulness in mechanical sampling lies in examining samples; in helping to interpret the samples in terms of dimensions, structures, water-bearing characteristics, and load-bearing capacities of subsurface formations; and in combining his interpretation of subsurface conditions with features of general and local geology to form the basis for recommendations for further sampling and testing."

The actual work—taking samples, testing physical properties, and most of the interpretation of physical tests—should be done by soil mechanicians and other qualified men.

It is interesting to paraphrase Mr. Feld's instance of the contractor who found himself in the class of an ex-contractor because he relied on a geological prediction that a certain hill was composed of glacial till, when excavation found solid rock instead: A patient died as the result of using drugs prescribed by a doctor who made an incorrect diagnosis of the patient's ailment. Perhaps the doctor was not very competent to diagnose that type of case, or perhaps his examination was not complete enough. In any event, such an example is scarcely a basis for condemning all doctors or the medical profession as a whole; nor does it indicate that it would have been preferable for either the patient himself or the druggist to diagnose the case and prescribe the medicine to be used.

Mr. Feld's experience undoubtedly has been shared by others. The cost of excavating solid rock would be less than that of excavating the type of earth encountered, but evidently solid rock was not preferred in the case Mr. Feld himself cited in the preceding paragraph of his discussion. In most instances, excavation of unconsolidated materials would be preferable; but the question is not what is more desirable but what will be found and in what quantities in a given excavation. The geologic history of the area will contain the answer, and the better the investigator knows that history the better equipped he is to find a solution.

In reply to Professor Legget's question as to the use of the term "traprock," admittedly the term should be avoided, and the writer's only excuse is that, in an earlier, unpublished, more detailed version of the paper, a table was included from a governmental publication giving the average results of physical tests on thousands of samples of various rock types. One of the rock types listed was "traprock." In the published version of the paper this table was not included, but inadvertently the term was retained in the text. For the purpose of this paper, "extrusive igneous rocks" should be substituted.

The "within limits" which Professor Legget questions should refer to the limits of the examining geologist's ability and the limits of geology in general, not to the limits of the depth and shape of the deposits.

Mr. Moneymaker partly answers his own question: "How should a state highway department provide for its geologic work?" Mr. Crosby offers further clarification. The answer depends upon the amount of geological investigation needed, the organization of the highway department, and the geological personnel and facilities available and economically practicable. These conditions vary widely from one state to another; each state has its own special problem.

The geologist who is to work with engineers when needed should certainly be an engineering geologist who devotes his full time and interest to that specialty. He cannot be primarily a petrologist or primarily a physiographer, for example, since geology like engineering is a highly diversified subject. Many geologists have specialized to such an extent that they have little

knowledge of, and less interest in, the geological fields not included in their own lines of specialization. The engineering geologist must be competent in most of the special fields and should have a wide range of interests and a broad outlook. His willingness to recognize his own limitations is extremely important. He must not overstep the bounds of his competency and should know when to call for the assistance of other specialists.

There are many geologists who think of their science as an end in itself. Obviously, such an attitude is unsuited to the practical field of engineering, where science is but a means to an end.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SURGES IN PANAMA CANAL REPRODUCED IN MODEL

Discussion

BY F. W. EDWARDS AND EDWARD SOUCEK

F. W. EDWARDS,[9] M. AM. SOC. C. E., AND EDWARD SOUCEK,[10] ASSOC. M. SOC. C. E.[10a]—It is interesting to note that one discussion expresses doubt regarding the ability of the model to reproduce surges that would be caused by operation of the proposed larger lock, whereas another expresses astonishment that it was considered necessary to conduct verification tests.

Mr. Weggel states, "Test No. III and Test No. V are of special interest because they illustrate the results of consecutive lock operations; but deductions are incomplete unless the data are available for the periods of synchronization and anti-synchronization." The data are available for these periods and were presented in Fig. 4.

In order to explain more fully, reference is made to Fig. 4(*a*). The initial lock operation depressed the water-surface elevation immediately. The second lock operation was performed 88 min later when the water surface was again approaching the low elevation. Withdrawing more water from the canal at this time lowered the water surface and increased the magnitude of the surge. It should be noted that 88 min after the second lock operation the water surface was low again but was not below the elevation caused by the initial withdrawal. The surge as augmented by the second operation, therefore, represents the worst condition likely to be produced by operation of the existing locks.

Reference is made next to Fig. 4(*b*). By performing the second lock operation 44 min after the first, when the water-surface elevation was approaching a maximum, the surface was prevented from rising to the elevation attained in Test No. III and the surge was neutralized. The choice of intervals between lock operations was not accidental, as implied by Mr. Leypoldt, but was based on calculation and on study of the water-surface profiles of Tests Nos. I and II.

NOTE.—This paper by F. W. Edwards and Edward Soucek was published in January, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1944, by Harold A. Weggel; May, 1944, by Harry Leypoldt; and June, 1944, by C. J. Posey.

[9] Associate Prof. of Civ. Eng., Pennsylvania State College, State College, Pa.

[10] Hydr. Engr., Office, Chf. of Engrs., Washington, D. C.

[10a] Received by the Secretary October 11, 1944.

Tests Nos. III and V substantiate Mr. Weggel's belief that, "Not always is it the amount of change of discharge that determines the height of wave or depth of trough." The data regarding waves on the Ohio River are interesting but do not support the opinion that a model could not reproduce, accurately, results identical to those observed in the prototype.

The writers agree fully with Professor Posey's viewpoint that comparisons of models and prototypes should be undertaken whenever practicable. Although it was expected that the model would represent the prototype correctly, it was considered good engineering practice to check results by every reasonable method available.

The entire discussion by Mr. Leypoldt is at variance with the beliefs of the writers. He states,

> "The water-surface profiles of Fig. 2 do not show the actual conditions. They are seriously influenced by the findings of Professor King, who states definitely that[3] the water is moving in an open canal when water is added or removed. In the prototype, as in all other seiches, the wave form is nearly sinusoidal, instead of the odd assortment shown in Fig. 2."

In spite of Mr. Leypoldt's belief that the wave form should be sinusoidal, actual data, as represented in Fig. 2, show this to be untrue in the present case.

The writers stated that the profiles should be considered pictorial rather than exact, but this qualification was made because a profile representing nine miles of canal is not precise when based on relatively few observation points. The statement was not made in order to justify a fictitious representation of the data. The wave forms represented in Fig. 2 were influenced only by the observed data; not by any literature, theory, or preconceived notion on the subject. The agreement between the water-surface profiles and the field observations for Test No. II can be seen in Fig. 6. In this figure the observed water-surface elevations from Fig. 3 and the profiles originally shown in Fig. 2 are plotted.

Fig. 6 shows clearly that the writings of Professor King did not influence the drawing of the profiles. However, the theories which Professor King has described are undoubtedly applicable to the generation of the initial wave. The relationships pertaining to the height of surges formed in an open channel by sudden changes in discharge are based only on the conservation of matter (implicit in the law of continuity) and Newton's Laws of Motion. The application of the equations is affected but not invalidated by the initial zero velocity of flow. It is well known that the initial shape of a wave in a channel is a function of the manner in which it is generated; whether the wave approaches regularity in form or tends to break up as it travels along the channel depends upon conditions beyond the scope of the paper. Whether or not the wave form is an "odd assortment" probably depends on the point of view. To the writers, any apparent oddity can be due only to an attempt to classify the wave forms in accordance with a preconceived notion—an error which Mr. Leypoldt attributes to the writers.

[3] "Translatory Waves in Open Channels," by Horace W. King, *Civil Engineering*, June, 1933, pp. 319–321.

The writers disagree with Mr. Leypoldt's contention that the data indicate a seiche of two or more nodes, one at station 1772 and another at station 1456. Since no significant oscillation occurs beyond station 1478, it is clear that none occurs at station 1456, but here the agreement ends. If a node were located at station 1772, the following conditions would exist: (*a*) The water surface at the

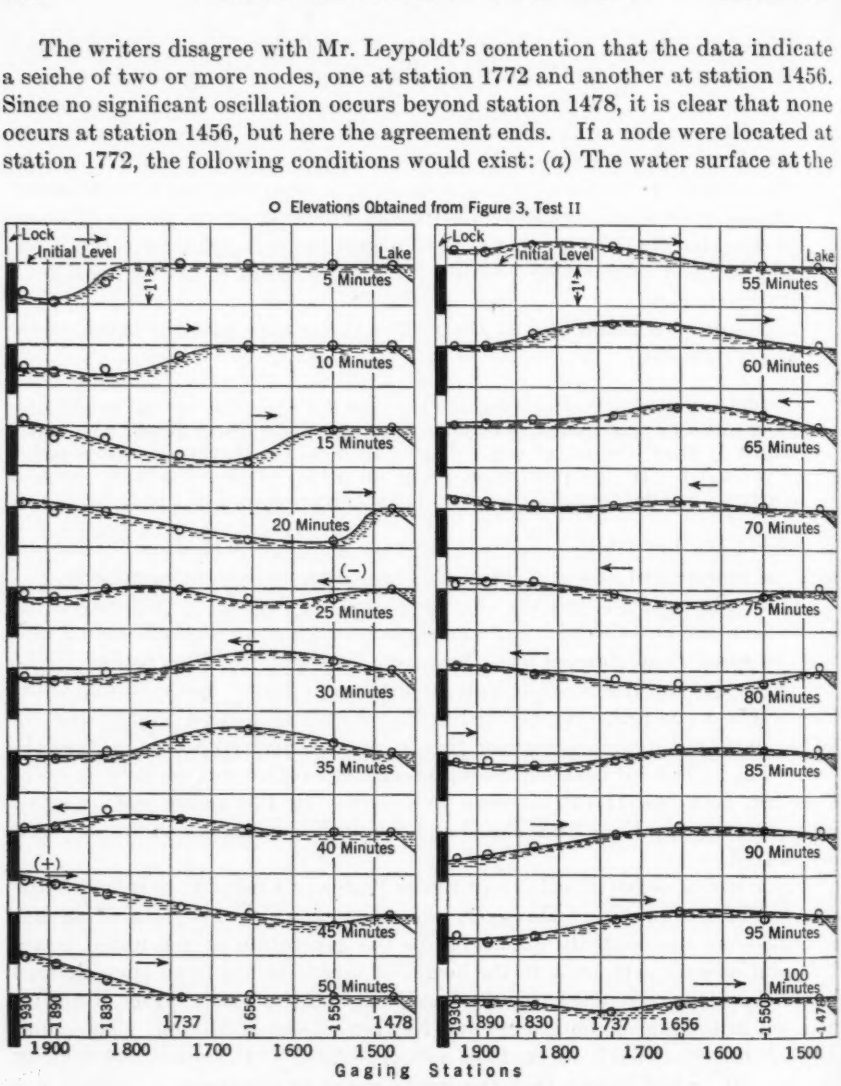


FIG. 6.—WATER-SURFACE PROFILES AT 5-MIN INTERVALS

node would oscillate less than at stations on either side, (*b*) the amplitude would increase progressively from the node to adjacent loops, and (*c*) the concurrent oscillations on the two sides of the node would differ in sign. The water-surface profiles of Fig. 2 show that none of these necessary conditions prevail for a node claimed to exist at station 1772.

Mr. Leypoldt believes the reference to Colonel Brown's work[4] to be irrelevant. He states that "Gatun Lake proper" does not influence the part of

[4] "Flow of Water in Tidal Canals," by Earl I. Brown, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), pp. 749–834.

the canal under discussion in the same manner that lakes do in Colonel Brown's studies. It is clear that Mr. Leypoldt means the main body of water approximately six miles from station 1478. However, Gatun Lake is considered generally and properly to extend from Gamboa, the point where the Chagres River flows into the canal at station 1478. The second paragraph of the paper and the notation of Fig. 2 testify that this designation was used by the writers. It is Gatun Lake at station 1478 to which the work of Colonel Brown is relevant and which does act in a manner identical to that of the lakes in his studies. It is Gatun Lake at station 1478 which was represented in the model by a forebay in which the water surface was maintained at a constant level. If the waves had not been reflected from station 1478 in the manner described by Colonel Brown, the model could have agreed with the prototype only by chance, the chance being the existence of a node at that station.

Since Mr. Leypoldt's seiche theory does include a node near station 1478, it is necessary to go one step further to show that Colonel Brown's theory of negative reflections does apply. This step is proof that significant oscillations do not occur beyond station 1478. Mr. Leypoldt suggests that the existence of additional nodes could be verified easily on the prototype by additional gages in the southern part of Gatun Lake. Preliminary reconnaissance prior to the tests showed that significant oscillations did not occur beyond station 1478, but to remove all possible doubt, a gage at station 1367+50 was established and operated during the first two tests.

The following quotation from the official report on the prototype tests[11] explains the abandonment of this station: "As had been expected, the surges beyond Gamboa were small so this station was discontinued after the second test." This gage was located near the point where Mr. Leypoldt's analysis indicates a loop.

The "narrow, shallow waterway except for the dredged cuts," and the Chagres River each have a mean width of more than one-half mile. The quiescence of these bodies of water during the surge tests shows conclusively that the bodies are large enough to reflect waves negatively in the manner described by Colonel Brown. The creation of significant oscillations in these waters by means of impulses no greater than those caused by filling the locks is physically impossible.

From Mr. Leypoldt's comments concerning Gatun Lake proper, from his opinion that no conclusions regarding model operations are valid from these experiments, from his statements that the width does not enter the problem and that roughness is unimportant, it is not evident that he understands that the model extended only to station 1478. The model did not encompass the length of the oscillating system which he believes to exist in the prototype. Accordingly the remark that it would be astonishing if the model were not a good replica of the prototype is especially inappropriate. Although it is true that all parts of the model could have been reduced proportionately in width on the basis of gravitational and inertial considerations alone, it is by no means evident that the rate at which the surges dissipate their energy would be unaffected. If

[11] "Surge Tests—Gaillard Cut, Report on Procedure and Results," by Edward Soucek, Third Locks Project; The Panama Canal, January 16, 1940, p. 3 (unpublished).

roughness affects the number of oscillations, which Mr. Leypoldt concedes, it must also have a progressive effect on their magnitude. Therefore, neither the width nor the roughness may be altered without careful consideration of the results. Since, in a model of the type described, many problems in addition to the period of oscillation are investigated—for example, the flow pattern at the junction of the by-pass for the proposed new lock—Mr. Leypoldt's suggestions regarding future experiments have no value. It does not appear desirable to adopt the restricted meaning of the term "surge," as advocated by Mr. Leypoldt, in a paper written for engineers.

Except to state that the specific suggestions offered by Mr. Leypoldt for solving the surge problems in the Panama Canal are considered impracticable, no comment on this question can be made at this time. The paper was specifically confined to an explanation of the surges and to verification of the model.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

MILITARY AIRFIELDS
A SYMPOSIUM

Discussion

BY L. J. MENSCH

L. J. MENSCH,[33] M. AM. SOC. C. E.[33a]—The student of highway and airfield pavements will gain much information from this excellent Symposium, especially if he compares the data presented by Brigadier-General Stratton with the specifications for concrete pavements available in the 1930's. This discussion will be confined mainly to the joint spacing and design of joints, as illustrated in Fig. 2.

In most cases longitudinal expansion joints may be dispensed with even in airfields 300 ft wide. Concrete pavements expand lengthwise from two causes—moisture variation and increase in temperature. The Arlington (Va.) tests of the Bureau of Public Roads (5*a*)[33b] show that, due to moisture variation alone, concrete pavements lengthen 0.0003 in. per in. in 4.5 years after construction. It is probable that the expansion may be as much as 0.0005 in. per in. after 10 years. The expansion due to an increase in the temperature of the slab depends on the difference between the temperature at which the concrete has set and the average temperature of the slab (between top and bottom surface) when the sun is shining on the pavement in June or July.

If this interval is 50° F, for example, the expansion from this interval will be approximately 0.00025 in. per in., or a total expansion of 0.0005 + 0.00025 = 0.00075 in. per in.; that is, 1.35 in. at each side shoulder in a pavement 300 ft wide. The shoulders will yield, and the only considerable resistance comes from the friction of the ground and the weight of the slab. Assume the slab to be 12 in. thick, weighing 150 lb per sq ft, and the coefficient of friction

NOTE.—This Symposium was published in January, 1944, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: February, 1944, by James B. Newman, Jr.; March, 1944, by Thomas E. Stanton; April, 1944, by W. E. Howland, and David S. Jenkins; May, 1944, by William E. Rudolph, Raymond L. Irwin, G. G. Greulich, Hibbert Hill, Jacob Feld, and Robert Horonjeff; June, 1944, by Clarence S. Jarvis, D. D. McGuire, E. D. Parmer, A. Casagrande, and Harry E. Cotton; September, 1944, by D. O. Murray, Thomas B. Pringle, John R. Haswell, W. J. Turnbull and William H. Jervis, Ralph R. Proctor, D. M. Burmister, Ralph Hansen, H. V. Pittman, G. R. Schneider, and H. M. Williams; and November, 1944, by Ira B. Mullis, and C. F. Izzard.

[33] Civ. Engr. and Contr., Chicago, Ill.

[33a] Received by the Secretary October 9, 1944.

[33b] Numerals in parentheses, thus: (5a), refer to corresponding items in the Bibliography at the end of the Symposium, and at the end of discussion in this issue.

to be 2 (5). The total resistance in a slab 12 in. wide, 150 ft long, and 12 in. thick will then be $150 \times 2 \times 150 = 45{,}000$ lb. This force divided by the section of the slab, 12 in. × 12 in., gives about 300 lb per sq in. as the maximum stress in the center line of the 300-ft-wide airfield. Such a compression helps the concrete as it diminishes any tensile stresses that may occur. Because of the time loading, it would shorten a pavement 150 ft long about 0.2 in. when the modulus of elasticity is assumed as 1.33×10^6, leaving a net increase in width at each shoulder of about 1.13 in. A drop in temperature of 100° F will shorten the 150-ft length about 0.0004 in. per in., or 0.72 in. This is expected to be distributed among the ten to fifteen longitudinal joints, so that there is a gap at each joint of from 0.048 in. to 0.072 in. Of course, if the dowels are not properly held in place, absolutely parallel to the direction of the movement (for example, if they should depart from the parallel as much as 1 in. in 16 in.), the dowels may freeze this movement in the joints and concentrate it in one joint. On the other hand, they may cause a longitudinal crack or at least a high tensile stress of the order of 300 lb per sq in. near the center of the pavement. When the designer has no confidence in the kind of load-transfer device he specifies or in the kind of workmanship he may obtain and when he fears that the dowels will not be parallel, then he is quite justified in providing a longitudinal expansion joint such as that shown in Fig. 2.

The size of the pavement slabs between the various longitudinal and transverse joints is shown in Fig. 2 to vary between 10 ft by 12.5 ft and 15 ft by 25 ft. It is generally believed that this size should be limited by the stresses generated by moisture and temperature variation. Another important point has been overlooked by General Stratton, however. The individual slabs are subject to rocking and vibration due to the heavy concentrated wheel loads at the edges and ends. These wheel loads must be resisted by the weight of the slab acting with a moment arm from its center of gravity to the part of the subgrade so located as to relieve the weight of the wheel load and its shock. Well-doweled joints will reduce this rocking greatly, and, therefore, dummy joints without dowels have no place in airfields. It is well known (5) that the greatest stresses occur at the ends and edges of a slab, and this is another important reason that joints in airfield pavements should always have load-transfer devices. No reliance can be placed on the interlocking power of concrete aggregate a few years after the pavement is built (5). Hence, the greater the wheel load, the greater will be the size of the individual slab. Possibly a size of 12.5 ft by 18 ft should be a minimum.

The fear that pavements may be injured by irregular cracking, due to warping from moisture and temperature variation, when the individual slabs are 10 ft by 25 ft or larger is based on the absence of an accepted theory concerning such actions and, of course, on everyday experience. In desperation some engineers have advocated slabs 10 ft by 10 ft which are entirely unsuitable for airfields. Slabs 25 ft long crack mostly because of faulty load-transfer devices, unequal frost heaving of poor subgrades, or excessive wheel loads.

Approximate Theory of Stresses Produced by Warping.—Temperature warping has been investigated thoroughly by engineers of the Bureau of Public Roads (5*b*), who found that the highest temperature differential, Δ, between

top and bottom surfaces occurs in the spring and early summer when the sun is shining on the slab. This value, in degrees Fahrenheit, is given approximately by

$$\Delta = 3.75\,d \qquad (18)$$

in which d is the depth of the slab in inches, corresponding to a unit differential in the length of these surfaces of

$$\delta = 3.75 \times d \times 0.0000048 = 0.000018\,d \qquad (19)$$

For example, a temperature differential of 33° F was found for a 9-in. slab. A weightless slab would deform into the shape of a sphere of radius $r = \frac{d}{\delta} = \frac{d}{0.000018\,d} = 55{,}555$ in. The Bureau of Public Roads measured the warping of a 9-in. slab, 20 ft long, at a temperature interval of 23° F. The radius in this case would be $r = 55{,}555 \times \frac{33}{23} = 79{,}700$ in. To simplify the calculation and at the same time to arrive at safe and acceptable results, assume that the warped surface is a cylinder with its axis along the transverse center line of the

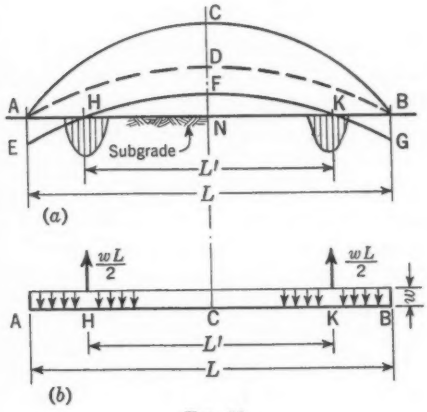


Fig. 52

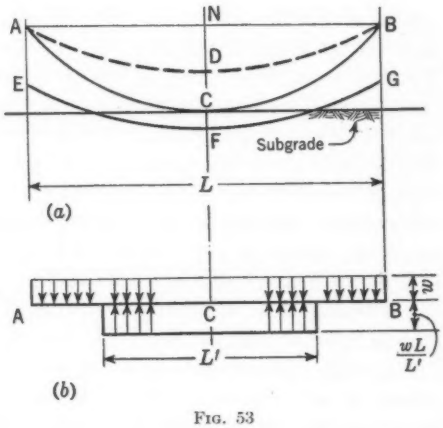


Fig. 53

slab, thus forming the arch ACB (Fig. 52(*a*)). The rise of this arch may be found by the well-known formula,

$$f = \frac{L^2}{8\,r} = \frac{240^2}{8} \times 79{,}700 = 0.09 \text{ in.} \qquad (20)$$

Under its own weight, however, the arch will become embedded in the subgrade near ends A and B, somewhat as shown by line EFG, depending on the soil modulus, the condition of the subgrade near the ends, and the moisture warping. The line ADB (Fig. 52(*a*)) has been drawn equidistant from line EFG to compare the deflection of the slab with the ideal condition shown by line ABC. The deflection CD is caused by the loading shown in Fig. 52(*b*); that is, by a uniform dead load of 113 lb per sq ft acting over the entire span of the arch

and by two subgrade reactions (shaded areas) assumed concentrated at points H and K, symmetrical with the center line.

The deflection caused by the uniform dead load is easily found to be: $\frac{w\,L^4}{8 \times 16\,E\,I}$. The deflection of the end in relation to the center point C caused by the load $\frac{w\,L}{2}$ at point H is most easily found by the moment area method: $\frac{w\,L}{2} \times \frac{L'}{2} \times \frac{1}{2} \times \frac{L'}{2}\left(\frac{2}{3} \times \frac{L'}{2} + \frac{L - L'}{2}\right) = \frac{1.688\,w\,L^4}{8 \times 16\,E\,I}$. The difference—$0.688\,\frac{w\,L^4}{8 \times 16\,E\,I}$—gives the deflection CD. Let $L = 240$ in.; $w = \frac{113}{12}$; $L' = 0.75\,L$; $E = 4 \times 10^6$; and $I = 729$. Then the deflection CD = 0.0578 in. The vertical distance between points E and F was measured as 0.035 in. (5*c*). This is equal to DN; and the total rise of the arch, CN = 0.0578 + 0.035 = 0.0928 in., as compared with the 0.09 in. found theoretically from the temperature interval. In other words, agreement between theory and practice is very good. The bending moment at the center of this arch, 1 ft wide, is found to be: $M = \frac{w\,L}{2} \times \left(\frac{L'}{2} - \frac{L}{4}\right) = 33{,}800$ in-lb, leading to a flexural stress of 209 lb per sq in. with tension at the bottom, as compared with a flexural ultimate strength in paving concrete of rarely less than 600 lb per sq in. Furthermore, this stress should be relieved considerably by the action in a transverse direction, which the writer has neglected, and by the continuity of good joints equipped with proper load-transfer devices. The Bureau of Public Roads estimated the largest temperature stress in this slab based on extensometer readings at 350 lb per sq in., but admitted that there might be some reservations concerning the assumed modulus of elasticity. For a differential of 33° F, a similar calculation and an assumption of $L' = 0.8\,L$ would result in a flexural stress of 250 lb per sq in.

Warping due to moisture variation was investigated for the Bureau of Public Roads by W. K. Hatt, M. Am. Soc. C. E. (61). He found the maximum differential change of length between the top and bottom surfaces of a wet subgrade on a dry summer day to be 0.000176 in. per in. in a slab 7 in. thick, in a beam 8 in. thick, and (it may be assumed) also for a 9-in. slab. A weightless slab would deform into a sphere, convex downward (see line ACB, Fig. 53(*a*)), with a radius $r = \frac{9}{0.000176} = 51{,}200$ in. and a rise (line NC) of $\frac{L^2}{8\,r} = 0.14$ in. for $L = 240$ in. Assuming again a cylindrical warping except that the dead load of the cylinder is supported by the subgrade in the middle part of the arch, these upward reactions may be considered to be nearly uniformly distributed on a length $L' = 0.6\,L$ without significant error. The deflection CD of the slab due to its uniform weight of w per square foot and the reactions $\frac{w\,L}{L'}$ per linear foot on the length L' may be found in a manner similar to that described for temperature warping: The deflection due to dead

load $= \frac{w\,L^4}{8 \times 16\,E\,I}$, when $w = \frac{113}{12}$; and the deflection due to the reactions $= \frac{w\,L}{L'} \times \frac{(L')^2}{8} \times \frac{1}{3} \times \frac{L'}{2} \times \left(\frac{3}{4} \times \frac{L'}{2} + \frac{L - L'}{2}\right) = \frac{0.408\,w\,L^4}{8 \times 16\,E\,I}$. The deflection CD resulting from time loading $= (1 - 0.408)\frac{w\,L^4}{8 \times 16\,E\,I} = 0.0982$ in., in which $E = 2 \times 10^6$. The measured vertical rise from the transverse joint edge to the center of the slab can be estimated (5*d*) at less than 0.04 in., giving the rise NC = 0.0982 + 0.04 = 0.1382 in. which is practically the same as 0.14 in. computed theoretically from length changes. The maximum bending moment per linear foot of width, $M = \frac{w\,L}{2}\left(\frac{L - L'}{4}\right) = 26{,}000$ in-lb, leading to a flexural stress of 168 lb per sq in. with tension on top, whereas the maximum stress from temperature variation was found to be 209 lb per sq in. at the bottom. Fortunately these stresses are not additive, except for the small effect of temperature warping at night.

The foregoing speculations seem to indicate that warping stresses increase somewhat less than the square of the length of the individual slabs, and, as the deflections increase with the fourth power of the slab length, soon a stage will be reached where the points D in Figs. 52(*a*) and 53(*a*) will reach the subgrade, which will probably occur in 10-ft-wide slabs when the length is about 25 ft; any increase of length will probably not increase the warping stresses to any extent. Much lower stresses occur in transverse directions at the center, and this fact should be taken into account when it is decided to use reinforcement steel to strengthen the slabs at the bottom. Reinforcing bars are advantageous if placed at the bottom in a longitudinal direction along the edges of the slab and in a transverse direction at the ends. Where top reinforcing is selected, the bars should be of high carbon steel and preferably ⅜ in. in diameter to avoid surface cracking. Such bars have been used by the writer with great success for pavements designed to support heavy trucks. General Stratton has treated the design of load-transfer devices rather meagerly. One gains the impression that he leaves to the discretion of the individual engineer the choice of positive holding devices or resilient fillers for expansion joints. The Bureau of Public Roads has shown (5) that ¾-in. dowel bars are not of much value for heavy loads, even if spaced 12 in. on centers. This has also been found true by the writer (62). It is quite correct to use a greater spacing of dowels for construction joints than for expansion joints, provided that such joints do not become expansion joints (as they have often been observed to become by the writer) while the expansion joints which have been provided become close. The writer has shown that dowels heavier than ¾ in. are needed for heavy wheel loads, especially such as occur on airfield pavements (62) and that it is of the utmost importance that the dowels be held positively, by other means than wire and stake supports, in the correct position parallel to the movement of the pavement. For airfield pavements with 150,000-lb wheel loads, dowels of large pipes or of similar sections will be needed, and the concrete around the dowels will have to be specially reinforced to prevent the

edges of the slabs from cracking due to the concentrated bearing pressures exerted by the dowels.

Bibliography.—

(5) **"The Structural Design of Concrete Pavements,"** by L. W. Teller and Earl C. Sutherland, *Public Roads*, Vol. 16, No. 8, October, 1935, pp. 145–158; Vol. 16, No. 9, November, 1935, pp. 169–197; Vol. 16, No. 10, December, 1935, pp. 201–221; Vol. 17, No. 7, September, 1936, pp. 143–171; Vol. 17, No. 8, October, 1936, pp. 175–192. (*a*) November, 1935, Fig. 14. (*b*) November, 1935, pp. 169–197. (*c*) November, 1935, Fig. 26. (*d*) November, 1935, Fig. 37.

(61) **"The Effect of Moisture on Concrete,"** by W. K. Hatt, *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 270.

(62) **"Joints for Concrete Pavements,"** by L. J. Mensch, Chicago, Ill., 1935.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

CHARACTERISTIC REDUNDANTS USED FOR ANALYZING STATICALLY INDETERMINATE STRUCTURES

Discussion

BY RALPH W. STEWART, MYLE J. HOLLEY, JR., M. W. HUGGINS, AND LEON BESKIN

RALPH W. STEWART,[10] M. AM. SOC. C. E.[10a]—The bending moments in the symmetrical portal frame analyzed in this paper (see Fig. 4) were computed from a full page or more of equations and formulas containing three X-values, two θ-values, one d-value, one κ-value, and nine Q-values. These moments can be obtained from two readings at one setting of a slide rule and a little easy mental arithmetic by sketching a traverse of the elastic curves of this frame (Fig. 8). This traverse diagram shows at a glance that three sevenths of the sum of the moments in either column goes to the top of the column and four sevenths to the bottom. From symmetry and the fact that the sum of all column moments must equal the lateral force multiplied by the frame height:

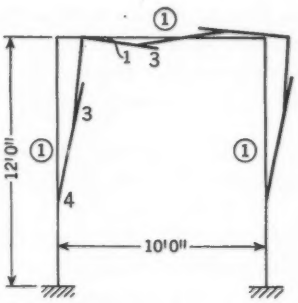


FIG. 8

$$\text{Moment at A} = 3/7 \times 6\,P = 2.5714\,P$$
$$\text{Moment at C} = 4/7 \times 6\,P = 2.4286\,P$$
$$\text{Total (check)} = 6.0000\,P$$

This one-minute solution uses two constants of flexure, one moment balance, and one traverse closure. The author's solution requires three simultaneous equations with sixteen symbols.

NOTE.—This paper by John B. Wilbur, Assoc. M. Am. Soc. C. E., was published in June, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1944, by Charles A. Ellis.

[10] Bridge Engr., City of Los Angeles, Los Angeles, Calif.

[10a] Received by the Secretary October 2, 1944.

Unless a type of problem can be presented for which the method of characteristic redundants is superior to other methods, the writer feels that the paper presents a method which has some value as a mathematical exercise but is not of value as a practical method of computing stresses.

MYLE J. HOLLEY, JR.,[11] ESQ.[11a]—A new method for the solution of statically indeterminate structures is suggested in this paper. In addition, an entirely new line of reasoning is introduced which may yield valuable concepts and methods beyond those illustrated by the author. Practical application of the method, along the lines suggested, should prove it more advantageous than conventional methods for some special problems, and, in addition, from such application may develop even more potent analytical methods. For this reason, it is to be hoped that some members of the profession will be encouraged to apply the method to practical problems.

The method of characteristic redundants for the solution of statically indeterminate trusses offers a reduction in the effort of solving simultaneous equations. This gain results from (*a*) the fact that only one set of simultaneous equations need be solved, regardless of the number of different loadings for which the structure is to be analyzed, and (*b*) the fact that there is one less equation in the simultaneous set that must be solved than there would be if a conventional approach were used.

The disadvantages of the method are that it requires the solution of a polynomial equation for the determination of characteristic forces, and that it also requires a substantially larger amount of tabular work for the determination of the participation factors. With these relative advantages and disadvantages in mind one might use the following criteria in deciding whether the author's method would be worth while in a particular case:

1. If the structure is indeterminate to a small degree, the new method will not prove advantageous, even if there are many loadings to be investigated.
2. If the structure is indeterminate to a large degree but only one, or at most a very few, separate loadings must be considered, the method will not prove advantageous.
3. If the structure is indeterminate to a large degree, and there are several separate loadings to be investigated, the method may prove advantageous.

The writer realizes that the foregoing criteria are very crude, but if the analyst follows some such reasoning before starting he may avoid an unfair test of the author's method and may also save himself considerable effort. Of course, only the experience resulting from actual analyses can refine these criteria to a point where they will give critical combinations of the degree-of-indeterminacy and number-of-loadings.

Although the writer can visualize no case in which it would be practicable to use characteristic redundants to determine deflections, it may be of academic interest to point out that this could easily be done. If the deflection of a point

[11] With S. Morgan Smith Co., Rutland, Vt.

[11a] Received by the Secretary October 13, 1944.

at which there is no load is desired, or if the deflection at a loading point in a direction other than that of the actual load is desired, the characteristic set would include a characteristic load at the point in question, in the direction of the desired deflection, as well as a characteristic load at each loading point in the direction of the actual load. The characteristic load at the deflection point will be removed by the participation factors in the superposition process which results in the actual loading. The writer believes, however, that such a procedure would be far more laborious than any of the conventional methods of determining deflections. This is in no way a criticism of the paper, which (as its title indicates) presents a method for analyzing indeterminate structures (not a method for finding deflections) and, for this purpose, the writer believes that the method holds real promise.

M. W. HUGGINS,[12] ESQ.[12a]—The skill displayed in producing this intricate and interesting solution to a complex problem is deserving of high praise. The aim of the author, which was to find what he calls "the heart of the multiply indeterminate solution" (see "Synopsis") and by means of which the analysis of any indeterminate frame is reduced to the solution of one system of simultaneous equations for all loadings, is a laudable one. Unfortunately, the solution obtained has not simplified the analysis of such frames. On the contrary, for most cases, it has added to the complexity of the analysis. It seems quite within the realm of possibility, however, that some future development of the method may serve to simplify the steps.

It would appear that the Wilbur method is in much the same category as the "relaxation method,"[13] proposed by Prof. R. V. Southwell. As an extension of the principles of moment distribution to the solution of multiply redundant trussed frames, Professor Southwell's method was first developed to simplify the analysis of dirigible frames. In it, stresses are determined by a series of approximations, but, unlike the method of moment distribution, the approximations do not converge readily. In fact, considerable skill and patience are required in solving even some of the simpler frames. In spite of this, the method seems to hold promise and has the important advantage that it eliminates, completely, the solution of simultaneous equations. It may be used to analyze any problem which normally involves the solution of simultaneous equations. As with the method proposed by Professor Wilbur, it has its greatest advantage when used to analyze one frame for several different, separate types of loading.

LEON BESKIN,[14] ASSOC. M. AM. SOC. C. E.[14a]—An attempt is made in this paper to apply to statically indeterminate structures a method of calculation which is highly successful in various fields of applied mathematics. The fundamentals of this method, as presented by the author in Sections 1 to 11

[12] Asst. Prof., Civ. Eng., Univ. of Toronto, Toronto, Canada.

[12a] Received by the Secretary November 2, 1944.

[13] "Relaxation Methods in Engineering Science," by R. V. Southwell, Clarendon Press, Oxford 1940.

[14] Structures Engr., Consoladated Vultee Aircraft Corp., San Diego, Calif.

[14a] Received by the Secretary November 8, 1944.

and 16 and 17, can be found in various good mathematical treatises;[15,16,17,18] the material in Section 16, however, lacks precision inasmuch as Professor Wilbur does not apply to his problem the classical properties of definite quadratic forms which constitute the basis of the method. This attempt to transpose into the field of statically indeterminate structures a method which is essential in other fields is not successful, due to the fact that the problems to be solved in the case of statically indeterminate structures are considerably simpler than those for which the method has been developed. Only in some special cases, most of which are already well known, does the method present definite advantages. Similar developments were presented in 1907 by S. Müller[19] and in 1944 W. M. Kincaid and V. Morkovin[20] presented a straightforward method of "orthogonalization" limited to the case of rigid frames.

To define the type of problem examined by the author, it must be remembered that the strain energy of a system is a quadratic function of redundants, and can be represented by an ellipse (two redundants), a quadric (three redundants), or a generalized quadric in a hyperspace (more than three redundants), these curves or surfaces corresponding to a determinate value of the strain energy.

The quadratic function is minimum at the center of the quadric, the coordinates of which represent the actual values of the redundants. The solution of a system of linear equations deduced from the condition of minimum strain energy is tantamount to the determination of that center.

Actually, the author examines two methods of determining that center. The first is the study of the principal axes of the quadric, which is of interest when these principal axes are needed as such—for instance, in problems of vibration, in which these axes represent the periods of vibration. When these quantities have no special interest, this method is much too complicated, since it requires the solution of an algebraic equation, the degree of which is equal to the number of redundants. The author has devoted more space to this method than it deserves.

The second method, which is specifically called orthogonalization of linear equations, is the study of an arbitrary set of conjugate axes of the same quadric—a much simpler problem than the first. Once these axes are defined, it is extremely simple to find the coordinates of the center, which are given by independent linear equations, containing one unknown each. This method, to be an improvement, must be shorter than the conventional method involving the direct solution of linear equations. It substitutes for the solution of n linear equations with n unknowns the solution of the sequence, $n - 1$ equa-

[15] "A Course of Modern Analysis," by E. T. Whittaker and G. N. Watson, Cambridge, England, 1927, p. 224 et seq.

[16] "Die Differential- und Integral gleichungen der Mechanik und Physic," by P. Frank and R. von Mises, F. Vieweg & Sohn, Braunschweig, 1930; American lithographed edition, Mary S. Rosenberg, New York, N. Y., Vol. I, 1943, Chapter II, pp. 44–119.

[17] "Methoden der Mathematischen Physik," by R. Courant and D. Hilbert, Julius Springer, Berlin, 2d Ed., Vol. I, 1931, Chapters I and II, pp. 1–50.

[18] "Mathematical Methods in Engineering," by Theodor von Kármán and Maurice A. Biot, McGraw-Hill Book Co., Inc., New York, N.Y., 1940, Chapter V, pp. 162–216; Chapter VIII, pp. 323–364.

[19] "Zur Berechnung mehrfach statisch unbestimmter Tragwerke," by S. Müller, *Zentralblatt der Bauverwaltung*, 1907, p. 23.

[20] "An Application of Orthogonal Moments to Problems in Statically Indeterminate Structures, by W. M. Kincaid and V. Morkovin, *Quarterly of Applied Mathematics*, Vol. I, No. 4, January, 1944, pp. 334–340.

tions, $n - 2$ equations, . . . 1 equation. A comparison of the number of operations shows that the method is advantageous only for four unknowns or less. For these small numbers of unknowns, the additional work required by the preparation of the method offsets the theoretical advantage, except when simple formulas can be established for all problems related to a given type of structure (for example, the centroid and principal axes of a beam section or the elastic center in a rigid frame).

Moreover, the most efficient provision for solving simultaneous equations is the Gauss method, a remarkably simple application of which was presented by P. D. Crout[21] in 1941. The Gauss method is a systematic method of eliminating unknowns,[22] which is at the same time a method of orthogonalization. This method is so expeditious that other methods are never justified except when special physical interpretations make them easier to understand, or when further computations require some intermediate steps which the Gauss method by-passes. A few examples are offered to demonstrate the special conditions under which orthogonalization along a definite physical pattern is useful and also to indicate when research involving the principal axes is justifiable.

The problem involving combined bending and direct load is a case in point. When moments are taken with respect to the centroid, the property of orthogonalization will appear; that is, moments then produce only rotation and direct loads produce only translation.

In the general case of bending, the principal axes of inertia have the property of orthogonality—namely, that bending around an axis produces rotation around that axis. In this case, principal directions are defined, thus requiring the solution of an equation of second degree, although, in the previous condition (combined bending and direct load), principal directions are not defined because of their lack of physical significance. In combined bending and direct load, an orthogonal system of the third order is defined.

A more complex case of orthogonalization, introduced in the analysis of open cylindrical shells (aircraft structures and cylindrical roofs of the Zeiss-Dywidag type), is that of torsion bending. When these structures are subjected to twisting forces, transmission of torque produces axial stresses, which can also be produced by externally applied direct forces with no resultant and no resulting couple. These direct forces constitute a mechanical entity which the writer has called a "bicouple."[23] The distribution of stresses under arbitrary forces is determined by a method of orthogonalization, which defines, successively, the direct resultant, two couples at the centroid (along principal axes), and a bicouple in reference to the shear center. The orthogonalization is of the fourth order in this case.

A well-known case of the third order is that of frames. This problem is generally treated by the use of an elastic center, but sometimes it is presented as a case of orthogonalization (complex bending and direct load), and is then called the beam-column analogy.

---

[21] "A Short Method for Evaluating Determinants and Solving Systems of Linear Equations with Real or Complex Coefficients," by Prescott D. Crout, *Transactions*, A.I.E.E., Vol. 60, 1941, pp. 1235–1240.

[22] "Methoden der Mathematischen Physik," by R. Courant and D. Hilbert, Julius Springer, Berlin, 2d Ed., Vol. I, 1931, p. 3.

[23] "Stress Analysis of Open Cylindrical Membranes," by Leon Beskin, *Journal of the Aeronautical Sciences*, Vol. 11, No. 4, October, 1944, pp. 343–355.

The writer has also examined the case of the two-story frame, leading to an orthogonalization of the sixth order,[24] which reduces to a fourth-order and a second-order problem, when the frame is symmetrical.

Incomplete orthogonalization is also used. The three-moment equation for continuous beams is an example: All moments at supports are orthogonal, except moments at three consecutive supports; the two-moment equation (conjugate point method[25]) is an improvement on the three-moment theorem, since only two consecutive moments are not orthogonal to each other. The method can be applied a second time, and new moments can be defined, which are entirely orthogonal to one another.

In all these problems, orthogonalization is justified because it has a physical significance. Incomplete orthogonalization is a compromise which greatly reduces the numerical work necessary to solve simultaneous equations, without introducing additional complications in the preparation of these equations. It is obtained by a rational choice of unknowns which, once established, does not require either any additional computation (continuous beams), or very little additional computation (centroid, elastic center), every time the method is applied.

Sometimes, also, the choice of unknowns facilitates handling equations by reducing the values of the rectangle terms (such as δ_{ab} in Eqs. 2) as compared with those on the first diagonal (δ_{aa}, δ_{bb}, $\cdots$). Then, the solution of the simultaneous equations is less sensitive to small errors in the determination of coefficients δ and in the subsequent numerical computation. This makes it possible to use a slide rule instead of a computing machine. The redundants are chosen in such a manner that the regions of the structure on which they act simultaneously are reduced to a minimum. For instance, considering a structure on elastic supports of various rigidities, it is inadvisable to use as an unknown the reaction of a very flexible support, which greatly increases all the coefficients of the simultaneous equations so that the coupling, represented by $\frac{\delta_{ab}}{\sqrt{\delta_{aa}\,\delta_{bb}}}$, is nearly equal to 1.

The foregoing comments show the broad advantages and limitations of orthogonalization. Details of the method presented by the author will next be examined.

As has already been indicated, Sections 1 to 13 are devoted to a method of determining natural frequencies. This method is also used, when the elastic stability of a structure is considered, to determine critical loads; these loads correspond to zero frequency. The author applies this method to the condition in which all masses and mass moments of inertia are equal to 1. Then the coefficients κ are the reciprocals of the squares of the frequencies ω:

$$\kappa_i = \frac{1}{(\omega_i)^2} \qquad (76a)$$

[24] "Rings for Two Decked Airplanes," by L. Beskin, *Aircraft Engineering*, London, England (publication pending).

[25] "Moments in Restrained and Continuous Beams by the Method of Conjugate Points," by L. H. Nishkian and D. B. Steinman, *Transactions*, Am. Soc. C. E., Vol. 90 (1927), pp. 1–206.

Eq. 76*a* shows that the set of conditions, expressed by Eqs. 1, is unduly restrictive, and that the actual set should be

$$\Delta_{ji} = \frac{\kappa_i}{m_j} P_{ji} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (76b)$$

in which m_j is a generalized mass at point j (mass if Δ_{ji} is a translation and P_{ji} is a force; and mass moment of inertia, if Δ_{ji} is a rotation and P_{ji} is a couple).

Then, n solutions are found in exactly the same manner as the author's solutions, the latter corresponding to $m_j = 1$. It is obvious that, when couples and forces are used simultaneously (Section 13), the condition $m_j = 1$ leads to coefficients κ which have indefinite units, since they are $F^{-1} L$ for translations and forces and $F^{-1} L^{-1}$ for rotations and couples. Thus, the author's solution is not homogeneous and changes with the choice of the unit length. For instance, a cantilever loaded at the end by a load P and a couple C has a deflection δ and a rotation ϕ:

$$\delta = \frac{P L^3}{3 E I} + \frac{C L^2}{2 E I} = \kappa P \ldots\ldots\ldots\ldots\ldots\ldots (77a)$$

and

$$\phi = \frac{P L^2}{2 E I} + \frac{C L}{E I} = \kappa C \ldots\ldots\ldots\ldots\ldots\ldots (77b)$$

which gives:

$$\frac{\kappa E I}{L} = \frac{1 + \frac{L^2}{3} \pm \sqrt{1 + \frac{L^2}{3} + \frac{L^4}{9}}}{2} \ldots\ldots\ldots\ldots\ldots\ldots (78)$$

If coefficients κ with zero dimension are chosen—for instance,

$$\delta = \frac{\kappa L^2 P}{3 E I} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (79a)$$

and

$$\phi = \frac{\kappa L C}{E I} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (79b)$$

—then:

$$\kappa = 1 \pm \frac{\sqrt{3}}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (80)$$

Remarks Concerning Solution of Eqs. 3.—Some of the statements in Section 16 must be examined. For example, concerning the first paragraph of that section, it is well known that the strain energy of an elastic solid is always a definite positive quadratic function, either of the forces or of the displacements. This means that all coefficients κ are real and, moreover, positive. In other words, all vibration modes are real. Thus, an elastic system of the type examined is always stable. This statement is true when the strain energy is of the form $\frac{1}{2} \Sigma(P \Delta)$. Then, the effect of the change of configuration under load is neglected, which is not the case when problems of elastic stability are examined. It is understood that such cases are not considered by the author, since he uses Eqs. 2 to define displacements.

For the case of equal roots Professor Wilbur's examination of κ is incomplete. When there are two equal roots, the determinant that yields κ can be reduced

to the second order by eliminating other roots, thus:

$$(\kappa - a_{11})(\kappa - a_{22}) - (a_{12})^2 = 0 \quad \ldots\ldots (81)$$

Eq. 81 has equal roots only if

$$(a_{11} - a_{22})^2 = -(a_{12})^2 \quad \ldots\ldots (82)$$

which requires that $a_{11} = a_{22}$ and that $a_{12} = 0$.

The physical significance of Eq. 82 is that the two corresponding frequencies must be equal. This will happen in any system, if masses are chosen in a correct manner, and will occur for a definite configuration when the dimensions have determinate values.

For instance, in Fig. 9, let a mass be placed at the elastic center, which coincides with the extreme end, E, of the beam. The deflections are:

$$\delta = \frac{P L^3}{E I} \frac{1 + \sqrt{2}}{3\sqrt{2}} \quad \ldots\ldots (83a)$$

and

$$\phi = \frac{C L}{E I}(1 + \sqrt{2}) \quad \ldots\ldots (83b)$$

If the radius of gyration of the end mass is $\bar{r} = \frac{L}{\sqrt{3}\sqrt{2}}$ ($\bar{r} = 0.486\ L$), the two frequencies are equal. By the author's method, the two values of κ coincide when $\bar{r} = 1$, making $L = \sqrt{3}\sqrt{2} = 2.06$. This value is independent of the unit of length. Thus, if the computation is in inches, the cantilever equal to 2.06 in. has the exceptional property that its coefficients κ are equal. If the computation is in yards, a cantilever 74 in. long has the same exceptional property. It is obvious that this is a characteristic that has nothing to do with the structure, but merely with the choice of units. This effect can be eliminated by choosing other units. In other words, the condition imposed by multiple roots is generally of no significance for the problem examined by the author, despite the fact that it has a physical significance in the problem of natural frequencies.

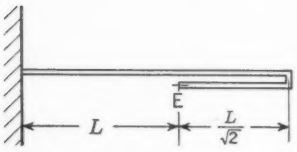


FIG. 9

When consistent units are used, multiple roots may have a physical significance. For instance, if the problem of general bending and axial force is examined in connection with a beam having equal moments of inertia in two directions, there are no principal directions; or, more exactly, all sets of mutually perpendicular axes can be assumed to be principal directions. It is simple and logical, in that case, to choose the principal axes of references as principal axes. The author examines two solutions (see Eqs. 59 and 60)—P_{a2}, $C\,P_{a2}$; and P_{a3}, $-\frac{1}{C}P_{a3}$—which define two perpendicular directions in the case of a beam. Factors C and $-1/C$ are the tangents of the angles of the principal directions with the initial axes. The logical choice in that case is P_{a2}, 0; and

0, P_{b2}. Then, instead of defining orthogonal systems by such complex formulas as Eqs. 60, the orthogonal system can be defined by:

$$P_{a2} = P_{a2}; \quad P_{b2} = 0; \quad P_{c2} = 0; \quad P_{d2} = -P_{a2} \ldots\ldots (84a)$$

and

$$P_{a2} = 0; \quad P_{b2} = P_{b2}; \quad P_{c2} = -P_{b2}; \quad P_{d2} = 0 \ldots\ldots (84b)$$

Characteristic Forces.—The first sentence in Section 13 is incorrect, unless the author has in mind a special definition of independent components of displacements. The latter are defined conventionally as the differences between the total number of displacements and the number of equations of statics. Moreover, for the example shown in Fig. 4, the author indicates (see third paragraph, Section 13) that there are three independent components of distortion, and it is well known that a built-in frame is redundant to the third order. Thus, the example is not consistent with the first sentence of Section 13, the meaning of which is obscure.

Alternate Basis.—The approach used in Section 17 is much simpler than that used in the remainder of the paper, and (as has been indicated) is the correct method for nearly all the problems considered here. It is correct to state that a judicious choice of arbitrary values simplifies the determination of dependent values; but simplification is generally obtained not by using values 0 or +1, as stated by the author, but by finding orthogonal groups from physical considerations, such as symmetry, antisymmetry, and polar multiaxial symmetry. In Section 14, the author examines such cases of simplification, but does not connect them with Section 17.

It is useful to clarify the connection between the two methods, seemingly independent in the author's presentation. This connection is very simple when the author's nonhomogeneous special relations (Eqs. 1) are replaced by Eq. 76*b*. Then, if a system is orthogonalized in an arbitrary manner by the second method, there will be a system of values m_i which will satisfy n systems of Eq. 76*b*, written successively for the n independent sets of values obtained by the method of orthogonalization. In Figs. 3 and 7, the values $1/m_i$ can be chosen equal to 1, 1, 0, and the values ω^2 can be taken as 1/100, 0, 1/100. In this case, there are indeterminate quantities 0/0, which can be defined by continuity from orthogonal sets differing from the solution selected, by infinitely small deviations.

These considerations show why the second method is simpler than the first: The system of masses m_i can have arbitrary values in the second method, whereas these masses are equal to 1 in the first. This determination of masses necessitates the solution of an equation of degree n, otherwise useless.

The advantage claimed in the last paragraph of Section 17 is purely fictitious. Experience in manipulating simultaneous equations reveals that, once the unknowns have been eliminated for a given set of loads (second members of the equations), the supplementary work necessary to find the redundancies in other loading conditions is extremely slight. When many loading conditions are contemplated (more than the order of redundancy), the use of unit conditions for the second members reduces the work in great proportions, and, if Mr. Crout's method is applied, this supplementary work is negligible.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

TRANSPORTATION OF SUSPENDED SEDIMENT BY WATER

Discussion

BY RALPH W. POWELL, AND E. R. VAN DRIEST

RALPH W. POWELL,[34] M. AM. SOC. C. E.[34a]—The subjects covered in this paper have recently attracted much attention, and the quite complete treatment the author has given is most welcome. It is hoped that in his closure he will explain the relation of his results to those of A. A. Kalinske and C. L. Pien,[35] Assoc. Members, Am. Soc. C. E., which seem to be founded on an entirely different approach, but which also lead to laws of logarithmic vertical distribution of both longitudinal velocity and sediment concentration. It may be stated, however, that G. I. Taylor,[36] in his "modified vorticity transport theory," deduces that velocity distribution for two-dimensional flow is cycloidal (except for the laminar boundary layer at the bottom). Although the cycloidal equation is entirely different from the author's logarithmic one, the numerical values differ hardly enough to be distinguished experimentally. The writer once amused himself by deriving a simpler distribution by thinking of the laminar two-dimensional flow of oil of high viscosity with a thin layer of water between it and the bottom. The velocity profile, of course, proves to be the parts of two parabolas, the lower one in the water being practically a straight line, and the upper one with its vertex at the upper surface of the oil. This gives an approximate picture of two-dimensional turbulent flow, the water representing the laminar boundary layer, and the oil the turbulent part with its high (mechanical) viscosity. It is only approximate because the mechanical viscosity is not uniform, but is a function of the depth.

Neither this rough approximation nor the more accurate cycloidal or logarithmic equations fit the case where the flow is not two-dimensional, however—as, for example, in narrow rectangular channels or rivers of nonuniform

NOTE.—This paper by Vito A. Vanoni, Assoc. M. Am. Soc. C. E., was published in June, 1944, *Proceedings*.

[34] Associate Prof. of Mechanics, Ohio State Univ., Columbus, Ohio.

[34a] Received by the Secretary October 4, 1944.

[35] "Experiments on Eddy-Diffusion and Suspended-Material Transportation in Open Channels," by A. A. Kalinske and C. L. Pien, *Transactions*, Am. Geophysical Union, Pt. II, 1943, pp. 530–536.

[36] "Flow in Pipes and Between Parallel Planes," by G. I. Taylor, *Proceedings*, Royal Soc. of London, Series A, Vol. 159, 1937, pp. 496–506.

depth. As is well known, in these cases the maximum velocity occurs some distance below the surface, and Eq. 15*a* could not possibly apply. The only satisfactory explanation of this depression, which the writer has seen, was developed independently in 1882 by the late F. P. Stearns,[37] M. Am. Soc. C. E., in America and Max Möller[38] in Germany, although Mr. Stearns acknowledges that the explanation had been suggested to him some six years earlier by the late Hiram F. Mills, M. Am. Soc. C. E. The explanation is that in straight channels there is a double spiral flow as shown in Fig. 16, superimposed on the

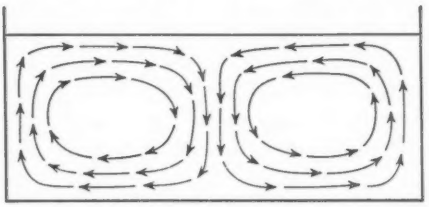


FIG. 16.—SECONDARY FLOW AS DESCRIBED BY STEARNS, MÖLLER, AND GIBSON

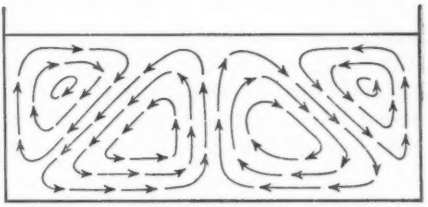


FIG. 17.—HYPOTHETICAL SECONDARY FLOW SUGGESTED BY THE WRITER

forward flow, which carries the slower moving water from the bottom up along the sides and out along the top to the center. At the center it mixes with what would otherwise be the fastest moving water, thus slowing the rapid water to a velocity somewhat less than that in the region just below.

Those who inquire as to what causes these spirals discover a very amusing situation. A. H. Gibson[39] follows out the thinking of Mr. Stearns and states:

> "Frictional losses at the sides reduce the energy and thus the head of the water in their neighborhood, with the result that the surface level at the sides is lower than near the center of the stream, and the cross-sectional profile of the water surface is a curve concave to the bed. Owing to this superelevation of the water near the center and its tendency to find its own level, transverse currents are set up which travel downwards near the center of the stream; outwards along the bottom to either bank, upwards along the sides, and, for permanence of *régimé*, inwards along and near the surface."

Mr. Möller,[38] however, takes just the opposite view:

> "Each filament of water that approaches the bank * * * suffers retardation, and hence requires greater cross section, whereas the conditions in the center of the stream are reversed, water filaments there being accelerated and requiring less cross section. Both actions lead to the production of a slight transverse drop in the water surface * * * and in the formation of spiral motion on either side."

Mr. Möller then shows an illustration with the water surface highest at the sides; but all three agree that the spirals are as shown in Fig. 16. Hydrodynamics would seem to the writer to show that these spirals would cause the water surface to be slightly higher at both the sides and the center, and lower

[37] "On the Current-Meter, Together with a Reason Why the Maximum Velocity of Water Flowing in Open Channels Is Below the Surface," by F. P. Stearns, *Transactions*, Am. Soc. C. E., Vol. XII (1883), pp. 331–338.

[38] "Studien über die Bewegung des Wassers in Flüssen mit Bezugnahme auf die Ausbildung des Flussprofiles," by Max Möller, *Zeitschrift für Bauwesen*, 1883, p. 201. A summary in English is given in *Hydraulic Laboratory Practice*, edited by J. R. Freeman, A.S.M.E. 1929, p. 70.

[39] "Hydraulics and Its Applications," by A. H. Gibson, D. Van Nostrand Co., 3d Ed., 1925, p. 331, following the article "On the Depression of the Filament of Maximum Velocity in a Stream Flowing Through an Open Channel," by A. H. Gibson, *Proceedings*, Royal Soc. of London, Series A, Vol. 82, 1909, pp. 149–159.

at the quarter points, but, since the velocity of the spirals is very small, the additional head due to curvature would be extremely minute, and at any rate it would be an effect, rather than a cause, of the spirals. The writer has the same feeling in regard to the author's conclusion (10), and believes that the double spiral flow is a cause of the unequal distribution of suspended load, rather than a result of it.

The cause of these double spirals is still obscure to the writer. L. Prandtl[40] has shown why there will be secondary flow toward the corners and away from the sides in the case of turbulent flow in rectangular pipes. Similar reasoning applied to a rectangular channel might explain spirals somewhat as shown in Fig. 17, but this is a pure flight of fancy on the part of the writer and he has no proof that it actually exists. There is just the possibility that such spirals might have been observed by Messrs. Stearns, Möller, and Gibson, and have been mistaken for the form shown in Fig. 16, but it does not seem very probable.

TABLE 5.—"EQUIVALENT ROUGHNESS" OF FLUME WITH VARIOUS SEDIMENT CONCENTRATIONS

Run No.	Number of values averaged	Average concentration, C_m (g per liter)	Mean diameter, $\overline{D}_s$ (mm)	Ratio of equivalent roughness to size of bottom sand
1, 2, 3	3	0	0	0.89
14, 20, 21	4	0	0	0.86
17	2	0.048	0.160	0.34
19	5	0.17	0.103	0.19
2	1	0.35	0.160	0.65
15, 16	2	0.59	0.160	0.40
1	4	0.72	0.160	0.78
4	2	0.79	0.160	0.89
3	2	0.85	0.160	1.02
18	4	1.18	0.103	0.40
20, 21	2	1.2	0.103	0.51
22	1	1.3	0.134	0.42
8, 14	3	1.34	0.160	0.68
9	2	1.41	0.160	0.36
5, 7	6	1.60	0.160	1.07
6	2	1.78	0.160	0.60
12	3	2.56	0.160	0.41
13	1	3.19	0.160	0.54
11	3	3.36	0.160	0.50

The other point which the writer wishes to discuss is the reduction in resistance to flow which the author finds accompanies the presence of sediment. The writer has suggested,[41] as a formula for resistance to flow in rough channels,

$$c = 40 \log \frac{R}{\epsilon} + c_s \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26a)$$

in which c is the Chézy coefficient, R is the hydraulic radius, ϵ is the equivalent roughness of the channel bottom (in the same units as R), and c_s is the shape

[40] "Über die ausgebildete Turbulenz," by L. Prandtl, *Proceedings*, 2d International Cong. for Applied Mechanics, Zurich, 1927, p. 62. The essence is given in English in "Fluid Mechanics for Hydraulic Engineers," by Hunter Rouse, McGraw-Hill Book Co., Inc., New York and London, 1938, pp. 267–268.

[41] "Flow in a Channel of Definite Roughness," by Ralph W. Powell, *Proceedings*, Am. Soc. C. E. (publication pending, December, 1944).

correction coefficient proposed by G. H. Keulegan[42] to correct the Chézy coefficient to a corresponding channel of infinite width; that is, to eliminate the effect of the side-walls. (The value of c_s is 14.18 times the value of β given by Mr. Keulegan.) In the author's experiments the side-walls were smooth and relatively far away from the sections studied, so that c_s may be taken as zero and R as y_m, the depth; thus:

$$c = 40 \log \frac{y_m}{\epsilon} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26b)$$

Values of c were computed for all the data of Table 3, and from these the value of ϵ, the equivalent roughness, given by Eq. 26*b*, was determined. These values were then divided by the diameter (in feet) of the sand forming the bottom roughness. For water without sediment it might be expected that this ratio would be 1, but the scale of roughness used in Eqs. 26 is not quite the same as that of Nikuradse, values of the writer's ϵ being somewhat smaller than Nikuradse's sand size.[41] All runs with approximately the same concentration were grouped together and the ratios of ϵ to size of bottom sand were averaged. Table 5 shows the results, arranged in ascending order of sediment size. The general weighted average for each size of sediment, irrespective of concentration, was as follows:

Mean diameter, $\overline{D}_s$ in millimeters	Ratio of equivalent roughness to size of bottom sand
0	0.87
0.103	0.32
0.134	0.42
0.160	0.69
All runs with sediment	0.59

Although it is evident that in general the equivalent roughness is reduced by the presence of sediment, there is great variation in the effect and there are even a few cases in which there seem to be an increase (runs 3, 5, and 7, Table 5). There also seems to be no relationship between the amount of sediment and the reduction, run 17 with only 0.048 g per liter giving the largest reduction of any concentration of the 0.160 mm sediment. The only consistent variation is that shown by the average for each size, which indicates that the reduction is greater for the smallest size and least for the largest. This might be interpreted as meaning that the reduction in resistance is a function of the number of particles present rather than of their total mass. It should perhaps be pointed out that the variation is not quite as great as might be thought at first. If the bottom sand were 0.47 mm in diameter and the depth 0.50 ft in both cases, the values of c corresponding to the ratios of 0.34 and 1.07 (the smallest and largest shown in Table 5 for 0.160 mm sediment) are 119.2 and 99.3, a variation of only 9.1% each way from the mean.

The writer has not tried to touch on some of the most interesting points in this valuable paper, which can better be discussed by those more familiar with the details of this subject, but has tried to indicate a few directions in which he hopes that further investigation will be made.

[42] "Laws of Turbulent Flow in Open Channels," by Garbis H. Keulegan, *Journal of Research*, National Bureau of Standards, Vol. 21 (December, 1938), p. 719.

E. R. VAN DRIEST,[43] Assoc. M. AM. SOC. C. E.[43a]—Experimental investigations conducted with the accuracy and thoroughness that is evidenced by this paper are of fundamental importance in dealing with problems of applied fluid mechanics. Particularly is this true in the field of sedimentation in flowing water where the number of variables encountered is great. The paper depicts clearly the trials and tribulations of an experimenter (1) in building apparatus for the proper control of conditions and (2) in bringing theory and data together.

In the presentation of the theory, it might add to the paper to point out the fact that the problem of suspension of sediment by flowing water is but one of the three important turbulence transfer problems usually encountered in engineering. The most common is that of internal friction or momentum transfer expressed by Eq. 2. Another such problem which has received considerable attention is that of heat transfer, the formula for which is

$$q = -\rho\, c_h\, \epsilon_h \frac{dT}{dy} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

in which, in addition to the notation of the paper, q is the rate of heat transfer across a unit area normal to the y-axis; c_h is the specific heat of the fluid; T is the temperature; and ϵ_h is the heat transfer coefficient. Through the assumption that $\epsilon_h = \epsilon_m$ (the basic assumption of the Reynolds analogy between heat transfer and skin friction), great progress has been made in heat transfer for flow in pipes. The third transfer problem—that of sediment transfer—has for its basic relation Eq. 4. The paper proceeds to show that progress is now being made in the sedimentation field when ϵ_s is assumed equal to ϵ_m or when the mechanism for the transfer of sediment is assumed the same as the mechanism for the transfer of momentum. Since these three phenomena are of the same nature, they can be derived from one general transfer formula,

$$Q_\theta = -\epsilon \frac{d\theta}{dy} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

in which Q_θ is the mean rate of transfer of any property θ per unit volume across a unit area perpendicular to y. By merely substituting any of the foregoing properties per unit volume—that is, momentum $\rho\, u$, heat content $\rho\, c_h\, T$, or sediment concentration C—into Eq. 28, the aforementioned equations will result. Indeed, any other properties may be assumed to be transferable in the same sense.

The author shows some concern about the fact that the velocity distribution data do not fall entirely on a straight line when plotted on semilogarithmic paper. Furthermore, he states that this departure is due to viscous action, but finds that such an assumption does not explain the results. The truth of the matter is that Nikuradse's data do not fall exactly on a straight line either.[44] Neither do his data for smooth pipes[45] fall precisely on a straight line

[43] Asst. Prof., Mech. Eng., Mass. Inst. Tech., Cambridge, Mass.

[43a] Received by the Secretary November 4, 1944.

[44] "Strömungsgesetze in Rauhen Rohren," by J. Nikuradse, *Forschungsheft 361*, Vereines Deutscher Ingenieure, Vol. 4, 1933, Figs. 15*a*, 15*b*, 15*c*, 15*d*, 15*e*, and 15*f*, p. 9.

[45] "Gesetzmässigkeiten der turbulenten Strömung in Glatten Rohren," by J. Nikuradse, *Forschungsheft 356*, Vereines Deutscher Ingenieure, Vol. 3, 1932, pp. 1–36.

when one plots the data separately for each run. All Nikuradse's data for smooth and rough pipes, except those for the lowest three Reynolds numbers in the smooth pipes when the viscous layer does show its presence, have a characteristic S-shape. A straight-line plot does appear to exist when all the data are plotted on the $\left(\frac{u}{\sqrt{\frac{\tau_0}{\rho}}}, \log \frac{y}{e}\right)$-scales, in[46] which e is the linear dimension characterizing roughness; however, even here the prevailing S-pattern can be observed for each run. That the viscous layer does not affect the author's data is evidenced by a simple computation which will show that the position of the data taken nearest the bottom is at least ten times as far from the bottom of the roughness grains as the edge of the laminar boundary layer which is at the most one half as thick as the sand roughness grains. Measuring the distance y from the bottom of the grains makes the measurements consistent with those of Nikuradse who measured his distances from the wall or, rather, from the base of the sand grains, and this does have a decided effect upon the plotting of the data very close to the wall. In Nikuradse's plots for individual runs, the straight line drawn for $k = 0.40$ passes through the data as a mean line. It appears that such a line for $k = 0.40$ has the same mean relation to the data of Fig. 8 for no sediment added, even after the data are shifted to conform with measurements from the bottom of the grains.

The data of Fig. 8 seem to prove quite conclusively that the value of k is decreased by the presence of sediment, and it appears logical that this should be so, because of the dampening effect of the sediment. However, the absolute effects of size and quantity are still uncertain, thus showing the need for further controlled experimentation. If the concentration has a decided effect upon k, then it should be logical to assume that the velocity distribution curve should be distorted considerably because of the large variation of concentration over the depth. This effect does not appear to exist.

An intriguing observation is that the exponent z_1 suddenly changes from the theoretical value when the size, and not the amount of sediment, is changed. The only explanation that the writer can give for this disagreement follows from the apparent circulation of sediment. Since the value of ϵ in any vertical line is more uniform near the wall than in the central part of the channel, the sediment transfer theory requires the sediment concentration to be more uniform vertically near the wall than farther in the channel thereby causing the concentration to be greater at the water surface near the wall than in the channel. This variation along the surface then causes a horizontal flow of sediment toward the center of the channel. In this process, however, all the sediment at the bottom corner is picked up, leaving a sediment gradient along the bottom decreasing toward the corner, which in turn results in a flow of sediment along the bottom toward the corner. The entire process obviously results in a circulation of sediment, not necessarily water, which may develop into a complex circulation pattern. It follows that heavier or larger grains should not be diffused too far into the flow and therefore that such sediment

[46] "Strömungsgesetze in Rauhen Rohren," by J. Nikuradse, *Forschungsheft 361*, Vereines Deutscher Ingenieure, Vol. 4, 1933, Fig. 14.

should be more uniformly distributed horizontally in the central part of the channel. On the other hand, the lighter or smaller grains would be diffused farther into the flow, thus easily causing higher concentrations near the surface at the central part of the flow and therefore producing values of z_1 smaller than the theoretical values. It is concluded, therefore, that complete sediment traverses throughout the stream are necessary to interpret, fully, the results obtained.

As mentioned by the author, sediment appears at the stream surface even though the momentum transfer coefficient there must become zero if the logarithmic velocity distribution law is assumed to hold all the way to the surface. This inconsistency promptly disappears when it is noted that the logarithmic law does not hold at the surface if the velocity gradient there is to be zero because of a presumably zero shear. Although one would expect the transfer coefficient to approach zero at the surface as a result of a zero approaching velocity gradient, yet some agitation should be expected because of the migration of eddies from the adjacent lower layer of turbulent fluid. Evidence of this diffusion can be observed readily on the surface of turbulent rivers. A more simple case of such diffusion is demonstrated by the movement of eddies from a point of disturbance at the bottom of a jar of water into the overlying quiet fluid.

The writer wishes to re-emphasize the need and importance of such basic experimental work in sediment transportation. The paper unquestionably represents considerable progress in this field.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

STABILITY AND STIFFNESS OF CELLULAR COFFERDAMS

Discussion

BY G. G. GREULICH, AND RAYMOND P. PENNOYER

G. G. GREULICH,[4] M. AM. SOC. C. E.[4a]—Some new methods, and an expansion of some old ones, for the precise mathematical evaluation of certain phenomena relating to cofferdam design are offered in this paper. The phenomena have been noted before, and empirical allowances have been made for their design and construction. The new concepts of analysis presented in the paper should be useful in providing additional cross checks for the adequacy of a cofferdam design. However, the use of precise mathematical evaluations can result in misleading, erroneous, and dangerous conclusions in the hands of inexperienced individuals, when dealing with such uncertain quantities as the widely divergent materials in ordinary soils whose properties and resulting physical characteristics and effects can vary widely due to either man-made or natural changes or disturbances.

Deflection of Cellular Cofferdams.—In cellular cofferdams, the nature of the deflection depends largely not only on the elastic properties of the fill in the cells, but also on the method of depositing the fill. For example, fill deposited in cells inundated in water will not produce the immediate tamping or packing effect obtained by ordinary methods of dry filling. Tightening the interlocks by removing loose play, as well as by stretching the entire cell, aids materially in reducing the deflection. The analogy is similar to the comparison of an empty headless barrel or keg with dried-out staves and loose hoops to one with tight hoops and staves in solid contact. Even without fill, the tight barrel will stand considerable lateral pressure at the top without appreciable deflection when the bottom is fixed. The loose barrel, of course, will rack far out of position. Where cells have been built and filled in deep water, the deflections under load are less for subsequent unwaterings than for the first unwatering because the consolidation and settling effect from the first drainage of the fill in the cells is still felt during later cycles of filling and emptying the coffer-

NOTE.—This paper by Karl Terzaghi was published in September, 1944, *Proceedings*.

[4] Cons. Engr., Specialty Div., Sales Dept., Carnegie-Illinois Steel Corp., Pittsburgh, Pa.

[4a] Received by the Secretary October 25, 1944.

dam. The deflection is also affected appreciably by the character of backfill material and the method of its deposition.

Permeability of Cellular Cofferdams and of Their Sheet-Pile Enclosures.—With reference to the permeability of sheet piling near the base of cells, tension in interlocks also prevails for a short distance below the base of fill. The softer the foundation material, the greater the depth of tension will be.

Resistance Against Sliding; Cellular Cofferdams on Rock.—Sheet piling is usually "toed in" on rock by driving so that it penetrates the soft upper layers of the rock. Where the rock is too hard to be penetrated by the sheet piling, mechanical methods such as dowels grouted into the rock have been proposed to prevent sliding. The method of grouting dowels into rock is highly objectionable due to intermittent supports and high stress concentrations at the points of support which would tend to rupture the sheet-pile interlocks. Another proposed method, which presents some very difficult construction problems, is the concreting of sheet piles in trenches cut into the rock.

Prior to the early 1930's, cofferdam design methods were based largely on "rule of thumb" and on precedent. The customarily accepted Pennoyer method (10)(11)[4b] was offered as a rationalized, simplified procedure, and not as a perfected, all-inclusive scientific approach to the problem.

As Professor Terzaghi states (see text concerning Eqs. 1 and 2), the Pennoyer design method disregards some factors that may reduce the margin of stability. When these factors are omitted, greater sliding resistance is indicated than is really the case when the actual shear distribution and height of saturation are taken into consideration. Inasmuch as there seem to be no sliding failures on record, the Pennoyer method has apparently been adequate, even though the factor of safety was not as high as computed. A more precise approximation can easily be obtained by applying the author's method. However, as Professor Terazghi states (see paragraph introducing Fig. 3), there is at present no reliable method of determining in advance the state and properties of the soil in the fill; and quantitative data on the stiffening effect of the sheet-pile walls are not available. Therefore, during the present state of the art, any precise mathematical analysis can at best be considered only an approximation.

Resistance Against Internal Shear; Cellular Cofferdams on Rock.—The author is in error in stating (see paragraph introducing Eq. 9) that possible failure by shear along the vertical neutral plane in the cells has never received any attention. Colonel Pennoyer has called attention to the investigation of internal shear resistance and its relation to the safety of the cofferdam (10*a*)(10*b*).

Resistance Against Overturning; Cellular Cofferdams on Rock.—The preceding remarks on resistance against sliding are also applicable to the factor of safety against overturning. Since the factor of safety of 3.8 suggested by the method of Colonel Pennoyer when applied to the selected example (see text following Eq. 20) is based on premises that may not actually exist, the factor with the assumed values, as asserted by Professor Terzaghi, is still about 2, which should be adequate. The writer presumes that in making this

[4b] Numerals in parentheses, thus: (10), refer to corresponding items in the Bibiography (see Appendix I of the paper), and at the end of discussion in this issue.

assertion Professor Terzaghi considered that the limitations imposed by internal shear are largely offset by the benefits of soil compaction and of friction in the interlocks.

With respect to overturning, fortunately the cellular type of cofferdam can be widened at practically no additional cost. Extension of the width makes the spacing greater between cross-walls so that the amount of steel is not increased in circular cells and is increased very little on the straight diaphragm wall type. In any case, the extra fill is a minor item. Of course, the allowable interlock tension may limit the width for a given height. In general, the cofferdam should be made as wide as reasonable limits allow. This increases stability and also helps in reducing permeability.

The increased stiffness of cellular cofferdams, as compared with other types, has been recognized in many ways, and lock friction (see second paragraph following Eq. 21) has been assigned an important place as a factor in increasing the stiffness of the cells. Colonel Pennoyer has mentioned this fact (10*b*). Heretofore, it has been difficult to evaluate the combined effect of the shear resistance of the fill and of the steel interlocks. It is recognized (returning to the loose barrel-stave analogy) that deflection or racking of the top of the cofferdam soon tends to increase the sliding resistance of one piece of sheet piling against the other in the cross-walls and that a large percentage of the total resistance to shear may be developed by the steel before the earth fill is distorted enough to contribute its complementary shear resistance. Therefore, the true value may lie somewhere between the value for earth fill alone and the combined value, as determined in the paper. Professor Terzaghi has proposed a method of approximating the combined values which appears to be simpler and more rational than any that has previously been brought to the writer's attention. The author's method, however, does not take care of another factor introduced by the sheet piling. The added factor is the direct tension introduced in the sheet piling when the cells cant or lean inward. This movement crowds the interlocks together so that individual pieces can exert tremendous resistance to sliding or upward movement. Because of these phenomena, the diaphragm or cross parts of steel sheet-pile walls are capable of transmitting some direct tension from the inside top of the cell, diagonally down to the outside bottom of the cell. In other words, the effect is similar to the introduction of stay rods from the inner top to the outer base of the cell.

The fact that interlock tension failures, as observed by Professor Terzaghi, usually occur before overturning moment is applied to the cells (see second paragraph preceding Eq. 23) seems to indicate that internal pressures and shear resistance in the fill are considerably more than the Rankine active pressure. This is probably due to the tamping effect of dropping fill in cells, as well as to a squeezing and consolidating effect occurring as the partly filled cells weave and stretch during filling and drainage of water. These factors all contribute to the ultimate stiffness and stability of the cofferdam.

There could be considerable danger of interlocks failing by tension at the inner toe of a cofferdam on rock if the piling were not toed or bedded in the rock. Only the most foolhardy individual would permit such construction, especially on wide cells. It is generally recognized that driving on rock tends

to damage the lower ends of the sheet piling and may produce incipient unlocking of adjacent piles. For this reason, manufacturers always recommend, and sound practice should dictate, the secure embedment of the lower ends on bare rock.

Even if the vertical reaction of the piles from overturning moment is theoretically sufficient to hold the piles in place as is repeatedly demonstrated by the shape of the bulge above the inner toe, the remarks regarding embedment of the lower ends still apply. Although, as Professor Terzaghi has emphasized (see third paragraph following Eq. 27), two of the three customary equations may be inadequate and the third is open to question, the fact remains that comparatively little trouble has been experienced with cellular cofferdams. As the author states, in the application of these equations, very conservative values have been used for soil constants. He expresses concern that an ambitious designer some day might use soil constants determined by laboratory tests, with fatal results. Fortunately, cofferdams usually represent the expenditure of considerable sums, and the final decisions regarding design and construction have been in the hands of mature, experienced individuals. The writer knows of no serious failures in which customary practices were followed by engineers, who generally allowed ample margins beyond the values obtained by the equations. Such practices may have been needlessly extravagant, and, admittedly, more exact methods of design are desirable to prevent future economic waste. There are also cases in which cofferdams with dimensions smaller than those indicated by the usual equations have been used without encountering difficulties; although in a few of such instances the factors of safety were so low that disaster could have followed any minor disturbance of soil or water level. In general, however, unanticipated forces or acts of nature have caused disasters which conceivably could have occurred regardless of any kind of theoretical analysis.

Penetration of Sheet Piling; Cellular Cofferdams on Deep Sand Strata.—Aside from consideration of uplift of sand due to percolation, it is believed that the depth for driving the sheet piling in sand need not be $\frac{2}{3}\,H$ (see second paragraph following Eq. 29*b*) but that $\frac{1}{2}\,H$ would be ample—because the sand in the cells can grip only one side of the piling, whereas, when driven, the piling is gripped on both sides.

Bearing and Extraction Values of Sheet Piling; Cellular Cofferdams on Deep Sand Strata.—Sheet piles in pairs may be driven and then loaded or extracted by means of hydraulic jacks to determine the forces required to shove down or pull out the piles (see second paragraph following Eq. 37). In similar soils, the values for a given depth of penetration are approximately uniform. It is easier to make an extraction or pulling test than a load test on the sheet piling, as anchor piles are not needed and the required pull may be exerted against cribbing on the ground. The bearing capacity may be safely assumed to be 15% greater than the extraction value, although, where piling is driven to rock, bearing value might be much greater than extraction value. The evaluation of vertical load capacities on sheet piling by means of hammer formulas is a very uncertain practice. The resistance of the interlocks when driving in a wall is a greatly varying factor—so great in some instances as to prevent

driving to moderate depths. The values for the sheet piles in the soil must be determined on piles separate from a continuous wall.

Conclusions.—The conclusions are most excellently stated. However, conclusion 2 needs further clarification and possible expansion. With respect to conclusion 5, the reduced pressures seem to be due to arching or cohesive action, or to a combination of both, since the Rankine theory is conservative and deals with granular materials only. In conclusion 6, the depth of penetration of sheet piling can be dictated by requirements of watertightness which, in some cases, may indicate a greater depth than is needed for load capacity.

Cellular Versus Double-Wall Cofferdams.—The author's comments about the superiority of the cellular, as compared to the double-wall, cofferdam are true only when full consideration has been given to adequate foundation support and the reduction of permeability in the supporting soil. Inasmuch as double-wall cofferdams usually have inner, and often outer, berms, their service record has been about as good as that of the cellular type. There have been failures of individual cells due to boils or undermining from percolation which might not have occurred on a wide-bermed, double-wall installation.

General Remarks.—Professor Terzagni has presented an analysis which should lead to further inquiring study and observation. He has provided another readily understood and practical series of steps for checking the design of any type of cellular cofferdam.

With this paper at hand, the designer may readily differentiate between an ultraconservative design for a permanent cofferdam or bulkhead and that for a temporary structure which, although it must be safe, may be allowed to have more deflection or movement while in use. If the movement becomes excessive in a temporary structure, it may be checked by one of the several known effective methods.

The author is to be commended for the thorough, thoughtful, and orderly presentation of a comparison between old and entirely new mathematical approaches on this subject. He deserves the wholehearted thanks of the engineering profession for assembling into convenient reference form much valuable historical and technical data on cofferdams, as well as for providing a most complete Bibliography on this difficult, complicated, and abstruse subject about which considerable quantitative information and experience must still be gathered.

Acknowledgment is made to J. R. Woods, sales engineer, Carnegie-Illinois Steel Corporation in Pittsburgh, Pa., for a constructive review of this discussion.

RAYMOND P. PENNOYER,[5] ASSOC. M. AM. SOC. C. E.[5a]—The basic computation to determine the necessary width of a cellular cofferdam, by bringing the intersection of the resultant of the moments created by two opposing forces (the lateral overturning force and the weight of the contents of the cells) within the middle third of the base, is expressed by Eq. 8. The author has stated that "This conclusion is open to serious objections." The only substitute offered in the paper for this basic determination is a statement that the

[5] Col., Corps of Military Police, U. S. Army (Inactive), Pittsburgh, Pa.

[5a] Received by the Secretary November 13, 1944.

"average width" of circular cells built on a rock foundation has been 0.85 H, in the past (see paragraph containing Eq. 27). All the remainder of the paper analyzes the theoretical internal stresses in a cell on the basis of this ratio.

Overturning Moment; Width of Cells.—The writer believes that 0.85 H has been more nearly the minimum width of completely excavated and unwatered cellular cofferdams than the average value. Furthermore, the proper and safe width of cellular cofferdams from which Professor Terzaghi has derived this questionable average has been determined, for years, by the very equation he states is open to serious objections.

It would seem axiomatic that, since the weight of the material contained in a cell determines its capability to resist the overturning moment, this material should receive careful consideration. Broken stone, gravel, sand, sand and clays, etc., have different weights, as have the undisturbed soils in which cells are built to various elevations. The analyses in the paper have all been based on sand weighing 110 lb per cu ft throughout the cell. No mention has been made of the possible buoyancy of water within the cells in so far as it will alter the weight of the contained soil (that is, the submerged weight); nor does Professor Terzaghi explain how he proposes to adjust the width of the cells to make the cells safe against overturning when there are almost infinitely variable factors affecting the weight of the contents.

No method is proposed to determine the width of cell necessary to resist the lateral or overturning forces safely. The paper deals only with an analysis of a cofferdam built in water on bare rock, with no submerged soil overburden. This condition is extremely rare in practice and always requires that sheet piles be driven into empty cells on the rock—a major construction problem. The width of a cofferdam in salt water should be different from that in fresh water, even under such conditions. Likewise, there must be an adjustment in width to resist the lateral pressures of submerged soils, for various elevations, in combination with layers of natural soil or fill above such submerged soil. There must also be an adjustment to resist other possible lateral forces due to construction equipment, storage loads, railroads, derricks, wave action, ice, swiftly flowing water, etc. This basic consideration has been dismissed in the paper by stating that the average width of cellular cofferdams in the past has been 0.85 H, and further that this ratio may be considered an acceptable basis for the design of cofferdams in clear water on rock.

Eq. 7, with its supporting text, is offered under the assumption that a cellular cofferdam filled with soil has a factor of safety of 3 when the width is calculated to bring the resultant of the forces within the middle third of the base. This formula is in serious error since the soil within the cells must be assumed incapable of resisting any tension or uplift along the outside face (the heel) of the cofferdam. The width of a cell must be adjusted to bring the stress to zero at this point (neither tension nor compression) and, as a consequence, the factor of safety against overturning is actually unity, which means that no factor of safety exists. Professor Terzaghi then discusses the possible, theoretical distribution of pressures under the cells (Fig. 7) and decides that, since a sand-filled cofferdam having a width of 0.85 H actually has its theoretical

factor of safety reduced from 3.8 to not more than 2, the internal shear becomes the critical consideration.

Internal Shear.—An attempt was made in 1934 to explain mathematically occasional leaning or distortion of the cells because of evident lack of internal shear resistance of the soil contained in the cells (10). At this time, the writer offered the following formula (10c) for vertical and horizontal shear, s, at a given point:

$$s = \frac{3\,P}{2\,b} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

in which, in addition to the notation of the paper, P equals the total lateral external load above the given point. Eq. 9, giving the total shear along the so-called neutral plane of the cell for a given moment, results in exactly the same value as Eq. 39 for maximum shear. Several successful bulkheads have been built which, upon analysis, were not more than 60% safe in internal shear, but which, at the same time, had a width sufficient to resist the estimated overturning based on the "middle third" method of determination. A mathematical theory of analysis will never serve as more than a guide based on precedent, because the assumption that the soil fails, along a plane of maximum shear, called the vertical axis, similar to a beam, seems fallacious.

What actually happens is that the entire mass of confined soil changes shape simultaneously. This action can best be described as a universal plastic flow, or distortion, which bears no relation to the strains set up in an elastic homogeneous mass that can resist shear, compression, and tension—with special emphasis on tension. When a cell leans inward, as a result of the simultaneous distortion, or flow, of all the contained material, the sheet piles in the internal, or diaphragm walls, remain parallel to each other and necessarily maintain their original length to lean into a parallelogram. Under this condition, all the interlocks in a cross-wall, or interior wall, slip the same amount at the same time, irrespective of their position in relation to the fictitious neutral axis.

As the writer stated previously (10), the resistance to lateral deformation of the bulkhead, through friction in the interlocks of the sheet piles, affords an indeterminate factor of safety in the calculation of internal shear. This still seems true despite the author's able mathematical analysis of this feature for a hypothetical cofferdam with a width of 0.85 H. With sufficient sand trapped and wedged into the interlocks, or sufficient penetration to develop frictional resistance against the sliding, the shear resistance of a cross-wall might be equal to that of a continuous plate of steel over the entire cross-wall. It is likewise possible that the assumption of the frictional value of an interlock as three tenths of the actual tension may be entirely too high. A cellular bulkhead built in undisturbed virgin soil, followed by excavation on one side, is a case in point. There might be no tension on the interlocks before the cells became distorted laterally.

Furthermore, the paper states (see sentence following Eq. 21) that the friction value f "is likely to be fairly close to 0.3" of the tension per unit of length on

a sheet-pile interlock—with no explanation as to the basis for this seemingly arbitrary assumption. If this factor was assumed analogous to the coefficient of friction of steel on steel, the statement would seem fallacious. Interlocks of sheet piling are designed for the least frictional resistance when setting up, driving, and extracting, with emphasis on reducing the area of contact between surfaces to a minimum and on eliminating wedging action from forces tending to separate the interlocks. There is no relation between the frictional value of two pieces of steel in what amounts to line contact and the coefficient of friction of steel on steel times a tension force per unit of length, when no area of contact can be assumed. In the discussion of the bulkhead in Fig. 11, Professor Terzaghi has expressed some doubt as to the safety of using three tenths of the tension for the frictional value of an interlock in his analysis of the actual distortion of the cells.

Tension in Interlocks.—The author recommends that the method of approximating the possible maximum interlock tension by the so-called Rankine formula, using $\tan^2\left(45° - \frac{\phi}{2}\right)$ to arrive at the conversion factor between the vertical and horizontal pressure, be abandoned in favor of using 0.4 when the cells are filled with sand and gravel. He states that this method does not create the illusion of resting on a scientific basis. It is seldom, however, that a cell entirely filled with sand is encountered in actual practice. Consequently, the paper leaves the reader without a practical basis for analyzing the problem that generally confronts him. As a result, use of the factor 0.4 might be worthless or even dangerous.

Professor Terzaghi indicates that designers seldom, if ever, use a conversion factor of much less than 0.4 for the lateral pressure of the contents of a cell filled with sand or gravel, completely drained and founded on rock. With a working stress of 4,000 lb per lin in. in tension for the interlock of the sheet pile, cellular cofferdams would be limited to a height of approximately 48 ft and a maximum radius of cell of about 24 ft. With a working stress of 6,000 lb per lin in., the limitations would be a height of about 58 ft and a radius of approximately 29 ft. Since there are many successful examples of bulkheads in service with dimensions considerably beyond these limits and containing soils far less ideal than the assumed sand and gravel, this conversion factor of 0.4 does not agree with actual practice. A conversion factor as low as 0.17 has occasionally been used without any indication that the lateral pressures in the cells were underestimated. Obviously, the relieving effect of wall friction, even to the point where a complete arch is formed as described subsequently, is an important factor to be considered when establishing the conversion factor for use in any specific instance.

The paper offers no suggestions for approximating the possible interlock tension: (1) When the cells are partly full of water and submerged soil (see "Part I. Practical Considerations"); (2) when the cells have a layer of clay or other soils at the bottom or at various heights as part of the original soil on which the bulkhead is built; or (3) when possible surcharge loads are present.

Another common condition not discussed in the paper is that in which the contained soil is not fill but is composed of various types of undisturbed soil into which the bulkhead is built before excavation on one side proceeds.

In his prior paper (10*d*)(10*e*), the writer illustrated a case in which external and internal earth pressures differ which included weep holes, submerged soil, and water levels at various heights. In the light of the practical problems illustrated and discussed, and considering the complexity of conditions encountered in practice, the writer feels that Professor Terzaghi has oversimplified the problem by assuming a cell width of 0.85 *H* and a conversion factor of 0.4 to develop the bursting pressures.

Probably no one realizes more clearly than the writer that, at best, the use of the Rankine conversion factor is a very approximate assumption. He will continue to use it, however, until something better is available, especially since water and submerged soil seldom are the critical considerations in overturning forces, weights of material, and bursting pressures. In the light of many precedents, the use of this method is a guide which helps the experienced engineer to determine the border line between conditions in which trouble with broken interlocks is likely to be encountered, on the one hand, and the likelihood that construction will be completed successfully, on the other hand. Mathematics is only one consideration in reaching this decision, because the final verdict must take into account the strength of the sheet piling as manufactured, precedent, the variety of conditions into which the sheet piling is set up and driven, and the experience, ability, and equipment of the constructer. Interlocks can be ruptured by sand trapped in the bowls during the pile driving; builders can fail to set and drive the sheet piling accurately; or the final closure pile can be used to pull the remainder of the wall into its proper place. In all such cases, indeterminate stresses are created in the steel, or an interlock is ruptured in the process. Certainly, mathematics is futile under such conditions.

Sand and gravel, or rock, may actually arch and form a complete bottom in cells above the base of the sheet piling. About 1932 the writer had occasion to inspect a bulkhead composed of a series of diaphragm-type cells, about 35 ft by 27 ft between cross-walls and 35 ft high, the sheet piling being driven into sand and gravel along a river bank. With the bulkhead in place, the sand and gravel on the river side were excavated to bare rock and used to fill the cells to the top. As excavation proceeded, it was found that the sheet piling had been driven through a layer of boulders to about 12 ft from the bottom and had been twisted and bent, the interlocks being ruptured in many places. Water coming through the bulkhead had washed away the original soil through the gaps in the piling up to the top of the breaks, some 12 ft off the rock, leaving three or four of the cells completely empty to about that elevation. The cells were standing, however, and were resisting the maximum lateral load from the river bank as intended. Above the 12-ft elevation, the sand and gravel had formed into about as perfect a dome arched between the side-walls, front-walls, and back-walls, as could have been built deliberately in brick or stone. There was an occasional sloughing off of sand in the dome, to be sure; yet, after the cells has been repaired, the superintendent reported, a week or ten

days later, that a bucket actually had to be used to break the dome, so that the fill could settle to the bottom.

An analogy may be found in a pail filled with water. Its resistance to overturning depends upon its weight, height, and cross section. All shear is taken by the steel shell of the pail as water has no shear or frictional resistance value. Other material, such as sand and gravel, does have a high shear and frictional value and consequently possesses the inherent ability to arch as described in the aforementioned example. Thus, the shear stresses in a steel bulkhead would be relieved by the soil or fill material, whereas in the pail of water the total shear would have to be transmitted by the walls of the pail without relief.

If the foregoing theory is correct in principle, it would indicate that perhaps the "middle third" limitations on width are conservative and do have a factor of safety because the "bottom" is actually able to resist some uplift at the heel. This may be one reason why mathematical analyses of internal shear seem at wide variance with the actual resistance of cells to leaning or distortion in some cofferdams. Calculations of interlock tension of moist (not submerged) soil would thus become a study in the theory of pressures in bins. This may account for the fact that the maximum pressure seems to be at the zone of greatest bulge in a cell, ordinarily about halfway up the height, although it has been observed that this varies slightly for different soils. The maximum bulge may occur at the level of the first actual "bottom." Of course, such a theory cannot be used when there is water in the cells, but the bursting pressure from the soil alone might safely be ignored under ideal conditions below the point of maximum bulge. It is difficult to imagine a condition for which the pressure of the water in the soil at the bottom could be critical, especially if the bulkhead had weep holes.

Furthermore, if the foregoing theory is correct, the distribution of pressures at the surface of the rock (Fig. 7) would differ from the true conditions, because all the pressure at the toe would have been transmitted to the steel by friction and arching. Observers have long noted that the overturning moment does not create any consequential added bursting pressure in the interlocks of the sheet piling because no consequential added deflection of the steel has been perceived. The arching theory suggested herein may account for that phenomenon also, since interlocked sheet piling is a fully supported column.

In general, it has been difficult for the writer to be completely satisfied with any of the theories as to why cellular bulkheads function successfully without adding the thought that arching of the fill actually forms a "bottom" or "bottoms" to the "pail" or cell. When a material like plastic clay or mud is used to fill the cells, the action becomes uncertain, variable, and completely different from the action caused by materials like rock, sand, or gravel. It may be that the incapacity of such plastic material to arch accounts for the difference.

The writer has watched the action of many cellular cofferdams since observing the sand dome in the broken cells as described herein, and as yet has found nothing to disprove his surmise. On the contrary, when such an actual bottom is assumed, the guides obtained from the application of the basic mathematical analyses are reassuring when compared with the characteristics and action of cofferdams in service.

Cellular Cofferdams in Clay.—In his former paper (10), the writer did not attempt to discuss cellular cofferdams founded on anything except rock, because he considered there were sufficient intangibles involved in that problem. Even with Professor Terzaghi's contribution, it would seem that the proper design of cellular cofferdams in clay (or even through overlying strata of clay or silt on top of stable strata), especially when submerged, is still largely dependent on making mathematics coincide with what experience, observation, and practice dictate to be adequate construction.

The determination of the proper width of a cellular structure, founded on a stratum of unstable soil, is still a problem largely dependent upon judgment based on precedent and experience. It is axiomatic that, as soon as the sheet-pile structure extends into soil, it becomes partly a cantilever structure. If it

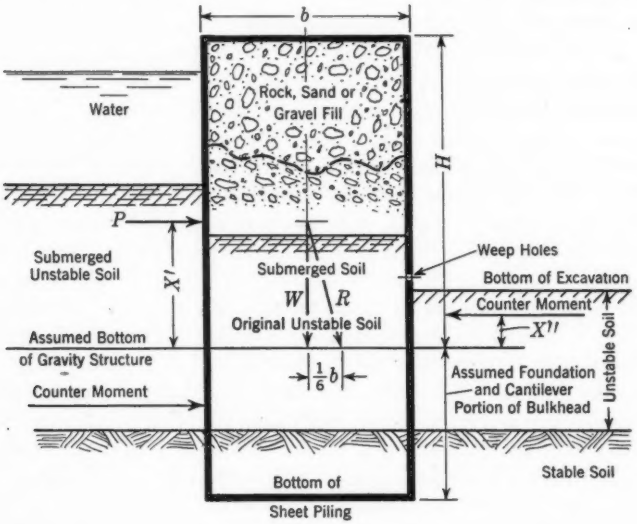


FIG. 12

extends deep enough and has insufficient width (weight of contents), it no longer is a gravity structure but a cantilever structure. So far as the writer knows, no attempt has been made, even theoretically, to design a cellular sheet-pile structure acting entirely as a cantilever. When all the intangible factors are taken into consideration, the problem would seem to be indeterminate.

Fig. 12 illustrates a design problem quite usual in practice, to the solution of which the paper has rendered little of tangible value. In this figure, in addition to the notation of the paper, R is the resultant of all applied forces, vertical and horizontal; and X' and X'' are moment arms as shown. Successful practice has indicated that a bulkhead can be considered as a gravity structure down to an assumed elevation, or "bottom." The selection of this fictitious bottom is a matter of judgment, taking all factors into consideration, including the resulting width of the cell. This elevation is not selected where the "counter-moment" due to the passive resistance of the soil balances the overturning

moment (because the calculated width of the cell would indicate a greater width than experience dictates is necessary) but at a somewhat higher elevation. One cannot take into consideration, either, the countermoment on the outside (heel) of the structure, because in this case the structure would be assumed capable of resisting the support stresses of a cantilever and there is no certainty that the structure is capable of transmitting the internal stresses properly. That part of the structure below the assumed "bottom" is thus considered as a foundation only for the gravity structure above this elevation, transmitting the compression forces due to the overturning moment and weight of cells into the soil below. Such resistance as that of this foundation—a cantilever against overturning, tension at the heel, and shear resistance against lateral distortion—may be assumed to be an indeterminate, but added, factor of safety.

The structure above this elevation of the assumed "bottom" is then analyzed for proper width, based on the width necessary to bring the intersection of the resultant of the forces represented by the weight of the contents of the cell and the overturning moment within the middle third of that bottom. The size of the cells must also be brought within a proper radius so as not to exceed the safe interlock tension in the sheet piles. Sliding is no longer a consideration and theoretical figures for shear resistance are balanced with precedent.

Professor Terzaghi states that the familiar "middle third" method of approximating the proper width of cell "is open to serious objections" (see sentences following Eq. 8); but he has proposed no alternate solution. The problem of how to determine the proper width of a cell under the aforementioned conditions, which are subject to variation on every project, has been dismissed with the very doubtful statement that the average width of such structures on rock has been 0.85 H and with the further note that internal shear is usually the governing factor. Obviously, the shear resistance of the contents of a structure under the aforementioned conditions could be much greater than the shear resistance of the contents of a structure founded on rock, which does not have the cantilever supporting value that the "foundation" gives to the sheet piling, with special reference to the internal walls. Eq. 32 giving the vertical maximum height H for a given bearing value of clay is of questionable practical value because the distribution of the pressure at the toe of the cofferdam, due to friction between the soil and the sheet piling, has been disregarded; the deeper the sheet piling is driven, the greater the distribution will be. Safe construction, as it concerns possible settlement at the toe, has always been assured when the sheet piling was driven to high resistance against further penetration, provided, of course, that there was no underlying unstable stratum below the bottom of the sheet piles.

After reaching the conclusion expressed by Eq. 32, the author proceeds to analyze mathematically the internal stresses in cellular cofferdams on a compressible base, using the "customary" values of $b = 0.85\ H$, etc., which, in Part I, he stated were the average ratios for sand-filled cells built upon rock. In practice, there is no relation between the two conditions; nor is the ratio of $b = 0.85\ H$ applicable, except by chance after the "middle third" method determines the proper width based on the assumed overturning moment and the weight of material in the cells.

Also, the author has analyzed the division of the total overturning moment on a cellular bulkhead (see Fig. 11) between the shear resistance of the fill and the sheet piling. He attributes the fact that the bulkhead began to deflect alarmingly (in spite of theoretical analysis) to the probability that the real lock tension (friction) might actually have been smaller than calculated. The writer doubts whether the actual overturning moment could be determined as accurately as outlined, since a few of the uncertain factors involved include the assumption as to lateral forces, the arm of the overturning moment or point of rotation, the fact that the structure is partly a cantilever and partly a gravity structure, the amount of cantilever support dependent on the lean of the cells, and the supporting value of the soil (both under and on the sides of the cells at the base).

Cellular Cofferdams on Deep Sand Strata.—Professor Terzaghi states (see "Part II. Design of Cellular Cofferdams") that a review of current methods of designing cofferdams reveals an "amazing situation," inferring that the seepage of water, possible boils in the bottom, and loss of stability of the foundation soil inside a cofferdam have not received consideration. The writer does not know of any phase in the design of cofferdams which has received more attention from designers and contractors. In 1935, the writer presented a discussion (29) of this hazard. This article offered some rule-of-thumb ratios between the hydrostatic head and depth of penetration necessary for the sheet piling in single-wall cofferdams when driven into various types of soil. These ratios were based on long observation of successful practice in preventing bottoms from boiling seriously or "blowing up." The ratio given for water-bearing sand was a penetration of 30% to 40% of the unbalanced hydrostatic head, provided the cofferdam is properly pumped, as against the 67% mentioned in the paper.

Also, the author recommends a penetration of $\frac{2}{3}$ H into sand as being necessary to prevent possible settlement at the toe of the cofferdam, due to vertical load on the sheet piles. It is commonly known that sand is probably the soil most resistant to the penetration of sheet piling and, under most conditions encountered in practice, the writer doubts if sheet piling could be forced into the depth of sand, indicated as necessary in the paper, without jetting, without rupturing interlocks, or without encountering absolute refusal. The writer has test results before him giving safe bearing loads of sheet piles, which show that a single pile tested to 47 tons when driven 25 ft into sand and when interlocked with two other sheet piles tested to 120 tons. When one considers that for even a 60-ft height of cofferdam the load on the sheet piles at the toe could hardly exceed 7 tons per foot width of sheet pile, $\frac{2}{3}$ H, or about 40 ft of penetration, seems at wide variance with the actual need.

In the discussion of filters and gravel toes for taking care of seepage, the author has omitted to mention the usual methods of stabilizing the soil inside cofferdams by pumping from sumps and the remarkable achievements of the so-called well-point systems in this respect.

The discussion of breakwaters founded on sand includes a recommendation that the sand should be compacted to a depth of at least 1.5 H in some situations. Under the conditions in Fig. 9(*f*), showing water going over the top of the breakwater, the critical consideration for stability is the fact that the

turbulent water scours the bottom on the harbor side until the breakwater fails, unless protected by a stone berm. Several articles on breakwaters published between 1932 and 1934 (30)(31)(32)(33) are enlightening with respect to the author's treatment.

Conclusion.—In general, the author deserves commendation for attempting to place the design of cellular cofferdams on a more precise mathematical basis. He has demonstrated that few problems have been overlooked in the past by the engineers and contractors who have had experience with the practical considerations necessary to design and build such structures properly and who have tried to reduce some of the intangible factors to some semblance of a precise basis for analysis. The paper is largely confined to a mathematical analysis of a cellular bulkhead of hypothetical size and weight of fill under a condition that is not common in practice. The attempt to apply the analysis, however, serves to indicate a wide variance between theory and practice. Dismissal of the old "middle third" method of approximating the proper width of a bulkhead under the wide variety of conditions met in practice by the substitution of the "average," "acceptable," or "usual" ratio of width equal to 0.85 H is not only erroneous but dangerous.

In estimating the interlock tension, a conversion factor of 0.4 for the computation of the lateral pressure in a sand fill would be too high under some conditions and dangerous in others. No method is proposed whereby the critical interlock tension can be determined or the maximum size of cells established to meet the wide variety of conditions encountered in practice. The conclusion that consideration of the shear value of the soil in cellular cofferdams is more critical than the determination of the proper width and maximum size of cells is fallacious.

Bibliography.—

(10) **"Gravity Bulkheads and Cellular Cofferdams,"** by Raymond P. Pennoyer, *Civil Engineering*, Vol. 4, 1934, pp. 301–305. (*a*) Eq. 13, p. 303. (*b*) p. 305. (*c*) Eq. 12, p. 303. (*d*) Fig. 4, p. 304. (*e*) Fig. 1, p. 301.

(11) **U. S. Standard Steel Sheet Piling,** Carnegie-Illinois Steel Corp. (Pittsburgh, Pa., Chicago, Ill.), 1st Reprint 1936.

(29) **"Design of Steel Sheet-Piling Cofferdams,"** by Raymond P. Pennoyer and George Hockensmith, *Civil Engineering*, Vol. 5, 1935, p. 19.

(30) **"Steel Breakwaters,"** by W. G. Caples, *The Military Engineer*, March-April, 1932, p. 120.

(31) **"Experience with Breakwaters at Milwaukee,"** by D. A. D. Ogden, *ibid.*, p. 146.

(32) **"Breakwaters on the Great Lakes,"** by W. F. Heavey, *ibid.*, November-December, 1933, p. 486.

(33) **"Developments in Steel Breakwaters,"** by W. G. Caples, *ibid.*, January-February, 1934, p. 39.

Corrections for *Transactions:* In September, 1944, *Proceedings*, on page 1016, change line 3 to read: "* * * cylindrical cells of the type shown in Fig. 1(*a*), interconnected by short arcs on the outer side of the dam. The * * *."

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

PRESSURE ON THE LINING OF CIRCULAR TUNNELS IN PLASTIC SOILS

Discussion

BY JOHN M. BIRD

JOHN M. BIRD,[7] ASSOC. M. AM. SOC. C. E.[7a]—This paper is an excellent interpretation of field data collected, for the most part, from measurements taken during and after the construction of the Lincoln Tunnel under the Hudson River at New York City. In addition to analyzing these data, Professor Krynine has compared the pressures on the Lincoln Tunnel with those on the Central Avenue Pipe Tunnel in Detroit. A few data referring to the Chicago Subway are also cited by the author. All three tunnels were constructed in plastic soil. This paper is a definite contribution to the extremely limited literature on earth pressures on tunnels. The writer feels that, in interpreting these field data, Professor Krynine has revealed a very important pattern that the earth pressures on tunnels constructed in plastic soils appear to follow. A study of the field data for the Detroit Tunnel indicates a pressure pattern similar, in part, to that of the Lincoln Tunnel. A study of the earth pressures observed on other tunnels seems to substantiate the existence of a definite pattern that the earth pressures follow.

The writer will discuss conclusions 1 to 7 in three parts. Conclusions 1, 2, 4, and 5 will be discussed in Part 1, conclusion 3 in Part 2, and conclusions 6 and 7 in Part 3.

Part 1.—The author states (conclusion 1):

> "During the construction of the Lincoln Tunnel, the weight of the overburden above the tube was relieved by the upward vertical shearing stresses."

This fact appears to be unassailable if the field data are taken at face value. The relief in pressure during construction and continuing for a certain short period after construction has been completed, as brought out in conclusion 4, is the first phase in the pattern followed by earth pressures on the lining. Ap-

NOTE.—This paper by D. P. Krynine, M. Am. Soc. C. E., was published in May, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1944, by M. A. Drucker.

[7] Asst. Prof., Civ. Eng., Univ. of Tennessee, Knoxville, Tenn.; on loan to Civ. Eng. Dept., Univ. of Texas, Austin, Tex.

[7a] Received by the Secretary September 18, 1944.

parently, there are two stages of pressure variation that must be considered in an analysis of data concerning pressure on the lining of a tunnel whether the tunnel be subaqueous or subterranean. The first stage is one in which pressures exist due to the action of a disturbed soil mass that is in a condition of temporary equilibrium. During the second stage, pressures exist due to the action of a soil mass which is undergoing readjustment in reaching a final condition of equilibrium. The first stage takes place during construction and for a short time following completion of construction. During this first stage, the vertical pressures on the lining are decreasing. Pressure-variation records for the Detroit Tunnel indicate that the vertical pressure on the lining decreased for a period of approximately three months. Similar records for the Lincoln Tunnel show that the average vertical pressure, after having been reduced to a specified standard loading, decreased systematically for the 253 days of observation "until it apparently assumed a constant value" (see conclusion 4). It is evident that the equilibrium of the earth mass was disturbed by driving the lining through the soil. Also, the soil mass is in a temporary condition of equilibrium during this stage since another variation in pressure, indicative of a readjustment of the soil mass, must occur before the final equilibrium is reached. Since long-time pressure observations were made at the Detroit Tunnel (from December, 1930, to January, 1941), it is possible, by examining the pressure variation during this period, to find evidence of the two stages of pressure variation. These records indicate that a decrease in the vertical pressure on the lining prevailed for a period of approximately three months (stage 1) and thereafter the pressure was constantly increasing for several years (approximately five years) until a final and permanent condition of equilibrium was reached (stage 2). Since, after this five-year period, the vertical pressure appeared to remain fairly constant, it is not unlikely that a final condition of equilibrium was reached. The period of observation for the Lincoln Tunnel is short, and, as the author states, no definite conclusions can be drawn from the information published to date. It is likely, however, that the period during which a steady decrease in the vertical pressure was noted (253 days) corresponds to the first stage of pressure variation and that a second stage during which the vertical pressures increase steadily will follow. In the case of the Detroit Tunnel, the vertical pressure increased until it nearly equaled the static head at the elevation of the measured pressure. Probably, a similar stage occurred in the case of the Lincoln Tunnel. The author states, however, that a final stabilization of pressure was reached at the end of the stage of decreasing vertical pressure. This seems unlikely to the writer.

As the author states in conclusions 1 and 2, the decrease in pressure is caused by vertical shearing stresses within the earth mass. It must follow that an increase in pressure, as was noted on the lining of the Detroit Tunnel, is due to the diminishing or vanishing of these shearing stresses. During the period of temporary equilibrium when the vertical pressures are decreasing,[8] "* * * maximum resistances in the mass are being mobilized to transfer weight away from the tunnel," and, during the period of increasing vertical pressures culminating

[8] "Earth Pressure on Tunnels," by W. S. Housel, published as part of the Symposium on "Earth Pressure and Shearing Resistance of Plastic Clay," *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p. 1045.

in final equilibrium, there is a transfer of some pressure to the tunnel. The writer believes that the soil mass, upon being disturbed, tends to form an arch above the tunnel, transmitting the pressure of much of the overlying soil through the soil arch to the earth mass adjacent to the tunnel. This idea was presented by the late J. C. Meem, M. Am. Soc. C. E., in 1907.[9] The two ideas, one of arch action within the soil and the other the relief in pressure through vertical shearing stresses, are merely two aspects of the same phenomenon. For subaqueous tunnels, it may be considered impracticable to assume that a definite relief in pressure will result from such arch action. However, in a material such as the organic clayey silt of the Hudson River and the blue clay in which both the Detroit and Chicago tunnels are built, a soil arch is probably formed. In the case of the Lincoln Tunnel, the creation of the bulge or the ridge at the top boundary of the earth mass during the initial period of the soil's disturbance appears to indicate that this arch action is initially in effect. It is unlikely that the soil arch will endure for a long period of time; in other words, it is unlikely that the vertical shearing stresses set up to relieve the pressure will continue indefinitely. In some way, due to some cause, the arch action will break down and transmit back to the tunnel lining some, if not all, of the load of the overburden (preconstruction pressure). The writer likens this breakdown of the arch action in the soil mass to the action that would take place within an asphalt beam that was initially deflected upward and allowed to remain over its end supports for an indefinite time. Although the beam would maintain this deflection for a certain period, eventually, due to its own weight and possibly due to vibrations from external causes, a form of plastic flow would cause the beam to return to its original horizontal position.

The nature of the stresses set up to relieve the vertical pressure on the tunnel lining is of great interest to the foundation engineer. Of much greater importance, however, is the necessity of answering the following three questions:

1. How much decrease in pressure can be expected from vertical shearing stresses, and for what period of time will this relief exist?
2. Why is not this relief in weight permanent, and during how long a period do the shearing stresses diminish?
3. Do the shearing stresses finally vanish or do they transmit, permanently, some part of the load from the tunnel lining to the adjacent earth mass?

Answers to these three questions will come only as a result of (1) many more field studies like those for the Lincoln and Detroit tunnels and (2) an interpretation of the data from these field studies like that in this paper.

To know fully how much decrease in pressure can be expected, more information as to the shape of the area on which the shearing stresses act must be secured. The straight-line shearing surfaces in Fig. 2(*a*) are but crude approximations. The exact shape of these surfaces is indeterminate and depends on the type and condition of the soil mass in which the shearing stresses occur. The period of time during which the relief in pressure exists is a variable dependent on local conditions. For the Detroit and Lincoln tunnels, the period

[9] "The Bracing of Trenches and Tunnels with Practical Formulas for Earth Pressures," by J. C. Meem, *Transactions*, Am. Soc. C. E., Vol. LX (June, 1908), p. 1.

lasted from three months to a year, approximately. The answer to the question as to why the arch action or the shearing stresses diminish lies, possibly, in the effects of vibrations set up as a result of externally applied forces. Even a slight vibration might cause sufficient readjustment of the soil particles to cause a redistribution of stress. The period of time for the transfer of load back to the tunnel is a variable dependent on local conditions. It may be a few months but, as was the case for the Detroit Tunnel, it may be a number of years. That some permanent or residual shearing stresses exist in the soil mass as a result of the disturbance to its equilibrium is indicated by the fact that the lateral pressures against the tunnel lining of the Detroit Tunnel remained fairly constant and were substantially less than the static head after final equilibrium was reached. W. S. Housel, M. Am. Soc. C. E., states[8] that an indication of a permanent change in the condition of equilibrium in the soil mass from its preconstruction state may be found in the fact that the tunnel structure weighed less than the displaced earth and yet, in final equilibrium, vertical stresses of approximately the same magnitude as the static head existed.

Fig. 10 shows the variation of the average vertical pressure with time. The writer has computed what may be termed the average static head by averaging the pressures due to the weight of the overburden at points O, D, C, and B in Fig. 3(*a*). A similar procedure was used by the author in determining the average p_d in Table 1. A value of 39.2 lb per sq in. is thus obtained for the average static head. A comparison of the average vertical pressure and the average static head should indicate that the corrected average pressure stabilized at 36.9 lb per sq in. on April 13, 1935, is still considerably below the average static head value of 39.2 lb per sq in. Probably some increase in the average vertical pressure has occurred since this date. It is possible that the pressure will become approximately equal to the static head, as was the case for the Detroit Tunnel.

The writer is unable to understand (see conclusion 5) why

> "The decrease in the average pressure on the lining of the Lincoln Tunnel * * * should indicate the gradual disappearance of the shearing stresses acting on the mass."

This decrease in pressure is indicative of the action of shearing stresses and only by an increase in pressure will the gradual disappearance of these shearing stresses be indicated.

Part 2.—The author states under the heading, "External Forces Acting on the Lining and on the Adjacent Silt Mass: Interpretation of the Curve $W_a - P_d$," that

> "Due to symmetry, there can be no horizontal shear and hence no vertical shear at point O (center line of lining). In other words, there can be no relief in the weight of the overburden at that point."

In conclusion 3, Professor Krynine states that

> "A diagram of shearing stresses for different points of the periphery, plotted from a horizontal line, has a zero ordinate and a horizontal tangent at the center line of the lining."

Unquestionably, due to symmetry, there can be no vertical shear at point O, but, if there is to be no relief in the weight of the overburden at that point, the rate of change of the shear $\frac{ds_s}{dx}$ is assumed to equal zero at point O. For example, if an element of the soil mass of unit thickness is taken directly above point O, as shown in Fig. 11, the shearing stress on a vertical section a distance Δx from point O will be $\frac{ds_s}{dx}\Delta x$. The pressure at the base of the element is p pounds per square inch, which corresponds to the measured p_d shown in Table 1. Writing the equation $\sum F v = 0$ for the forces on the element:

$$\gamma h \Delta x - p \Delta x - \frac{ds_s}{dx}\Delta x h = 0 \ldots\ldots (3)$$

and

$$\gamma h - p = W_a - P_d = \frac{ds_s}{dx} h \ldots\ldots (4)$$

in which the weight of the overburden expressed in pounds equals $\gamma h \Delta x$; and γ = unit weight of the mass.

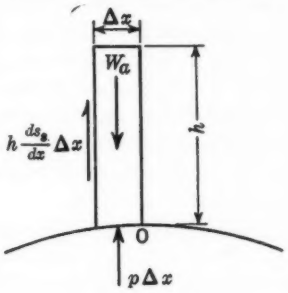


FIG. 11

To have no relief in weight at the center line, the rate of change of the shearing stress $\frac{ds_s}{dx}$ must equal zero at point O. The author apparently makes this assumption in conclusion 3 by stating that the diagram of shearing stress has a horizontal tangent at the center line. The writer admits that this may be the case. Evidence of such a condition will come only through experimental study. However, it does appear that the field data fail to support this theory, provided symmetry is assumed. A study of Fig. 7 shows that, on July 8, 1935, there is an overload at point O (P_d being greater than W_a) and also that downward vertical shearing stresses exist, although the values of these stresses have been omitted from Fig. 7. The writer has drawn the complete ($W_a - P_d$)-curve for this date and has determined the downward vertical shearing stress at point O in an approximate fashion (see Fig. 12). In making this rough computation of the vertical shearing stress at point O, the writer followed a method similar to that used by the author in his determination of the shearing stresses at the other points on the periphery. The area under the ($W_a - P_d$)-curve between the zero point (4.8 ft to the left of point O) and point O was computed on the basis that the curve was parabolic. This area, represented by $2/3 \times 4.8 \times (1.6 \times 144)$, equals 740 lb—the accumulated relieved weight between the two points in question. The area of action of the shearing stress is 28 sq ft, using the value of 28 ft for the height of the silt column above point O in Fig. 3(*b*). The approximate downward shearing stress equals $\frac{740 \text{ lb}}{28 \text{ ft}}$ or 26.4 lb per sq ft. On July 8, 1935, the vertical downward pressure P_d equaled 39.1 lb per sq in., the weight of the overburden W_a equaled 37.5 lb per sq in. (showing

an overload of 1.6 lb per sq in.), and the vertical shearing stress (downward) was approximately 26.4 lb per sq ft. Fig. 7 shows that, on July 16, 1935, and on August 8, 1935, the weight of the overburden was relieved by the action of upward vertical shearing stresses. The field data show that, on August 29, 1935, no relief in weight, and thus no shearing stresses, existed at point O. Accepting the field data as exact, it must follow that on the first three dates symmetry was lacking and the rate of change of the shearing stress $\frac{ds_s}{dx}$ was not equal to zero at point O. If pressure intensities had been taken at corresponding points on each side of the center line of the lining, it would have been possible to determine if symmetry did exist. Reviewing Table 1, downward pressures, p_d, appear to have been taken only at points O, D, C, and B on the periphery; thus, it is impossible to tell whether there was a symmetrical condition about the center line on the aforementioned four dates. The deformations arising from the decrease in the horizontal diameter of the lining that commenced as the lining was being driven through the silt may have caused the symmetry to be broken.

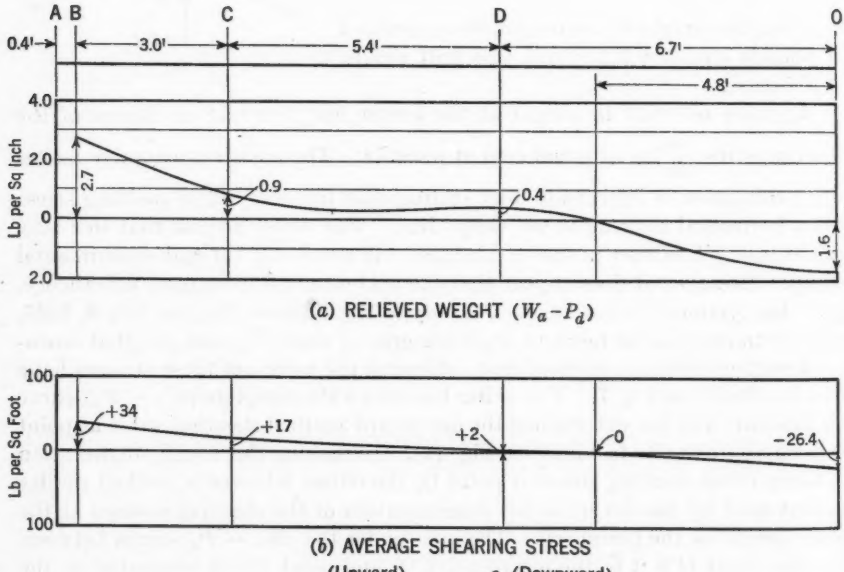


FIG. 12.—COMPLETE ($W_a - P_d$)-CURVE FOR JULY 8, 1935

Part 3.—The decrease in the horizontal diameter commencing immediately after the erection of the lining is an extremely interesting phenomenon. The undeniable implication arising from this occurrence is that the horizontal pressure on the lining must exceed the vertical pressure. Such a relation between the horizontal and vertical pressure is not commonly expected in an undisturbed soil mass. The ratio has been found to be approximately 0.7 for plastic soil of

the type of the Hudson River silt.[10] It appears logical that this phenomenon should occur only in soils that have a considerable ratio of the horizontal pressure to the vertical pressure in the mass. Since the field data for the Lincoln Tunnel show that the ratio of horizontal pressure to vertical pressure is less than unity (0.74 on August 29, 1935), an explanation of this phenomenon is necessary. The writer feels that the author's explanation—namely, that its cause must be sought at the very beginning of the shoving—is a probable answer.

An examination of Fig. 9 showing the horizontal pressures on the tunnel lining indicates to the writer that the general shape of the curve from point O to point B′ (P to N) follows the straight-line law (triangular pressure distribution) as used by Rankine in his formula. This would lead to the conclusion that the Rankine formula may be applied without serious error to the determination of lateral pressures from point O to point B′. Conclusion 7 is excellent. It is logical that the Rankine formula should not be used for determining lateral pressure below point B′ since the pressure on this section of the lining is shown by Fig. 9 to be fairly uniform.

General Remarks.—Great care should be exercised in interpreting field data such as those for the Lincoln Tunnel. Fig. 7 shows that the maximum relief in weight, occurring at point B on the periphery, ranges from 2.7 lb per sq in. on July 8 to 5.8 lb per sq in. on August 29. Also, at the center line of the lining (point O) the "relief in weight" varies from an overload of 1.6 lb per sq in. on July 8 to a load of O lb per sq in. on August 29. In view of the fact that the accuracy of measurements made with the special silt gage and plugs was limited to the range, 1 lb per sq in. to 2 lb per sq in., it is entirely possible that much of the difference in W_a and P_d may have been caused by the inability of the gages to record true pressures. Certainly the overload at point O on July 8 could be attributed to this cause. It is within the realm of possibility that there was no relief in pressure. Although the writer acknowledges that this is not likely, he merely wishes to state that a slight inaccuracy in the pressure-gage measurements could easily account for a difference in pressure of 2.7 lb per sq in. In Fig. 3, the author presents the computed values of the overburden pressure W_a at ten points on the periphery, using in these computations a value of 64 lb per cu ft as the density of the Hudson River water and a value of 104 lb per cu ft as the weight of the saturated clayey silt. An error of as little as 4 lb per cu ft in estimating the density of the silt could produce a change in pressure on the lining equal to some of the values considered in this paper as relief in weight. To illustrate the effect of a variation in density of the silt on the lining pressure, a column of the silt mass 20 ft high and 1 sq in. in cross section, with a volume of 240 cu in. or 0.14 cu ft, has been selected. A variation in density of 4 lb per cu ft would cause a change in pressure of 0.56 lb per sq in. at the base of this column. Such an error in determining the density of the silt mass is certainly possible, since there is doubt in the writer's mind that a homogeneous, uniform material exists for 20 ft below the bed of the Hudson River. In addition, it does not seem likely that the bed of the Hudson River is a level ground surface.

[10] "Design of Tunneled Sewers in Unstable Ground," by Robert G. Hennes, *Civil Engineering*, April, 1932, p. 248.

Therefore, it is difficult to justify using values for the height of the silt columns above the tunnel lining expressed to the nearest 0.1 ft. Certainly field measurements of this accuracy would be most difficult to make in the near fluid silt. A slight variation in the height of the overburden could cause substantial changes in the values of W_a and could be responsible for some of the difference between W_a and P_d hitherto attributed to relief in weight. Inaccuracies in gage readings, errors in estimating the density of the silt mass, and errors in determining the height of the silt column above the tunnel lining could produce results giving a pressure difference less than that which actually existed. The only purpose in emphasizing these possible sources of error is to caution against too strict an interpretation of the field data. Unless these data are exact (a fact which any engineer familiar with field and laboratory measurements will doubt), misleading results may be obtained. No criticism of the author's valuable interpretation of the field data on the Lincoln Tunnel is intended.

This excellent paper is a real contribution to the literature on earth pressures on tunnels. Many more similar studies are needed before definite conclusions can be reached as to the variations in pressure on tunnel linings constructed in plastic soils.

Corrections for *Transactions:* In May, 1944, *Proceedings*, on page 632, change line 21 to read "* * * order. If two bodies in contact tend to move along the plane of contact with respect to each other, the force * * *"; on page 637, change line 22 to read "(* * * lining as in Fig. 1(*a*)), P_d and P_u are forces computed from * * *"; and, after line 41, page 637, add the following new paragraph:

"In Fig. 5 and in all subsequent figures and discussion capital letters designate total forces acting on a slice 1 ft long cut from the lining by two vertical planes perpendicular to its longitudinal axis. These forces are represented by the areas of corresponding diagrams (for instance, areas under curves W_a and P_d in Fig. 7). Unit pressures (or shearing stresses) acting on the lining in vertical planes parallel to its longitudinal axis are ordinates of the corresponding curves. The use of lower case letters for unit pressures was thus avoided except in Table 1."

ENGINEERING
LIBRARY

DEC 23

PROCEEDINGS

American Society
of
Civil Engineers

DECEMBER

1944

VOLUME 70 NO. 10

AMERICAN SOCIETY OF CIVIL ENGINEERS

OFFICERS FOR 1944

PRESIDENT

MALCOLM PIRNIE

VICE-PRESIDENTS

Term expires January, 1945:

EDGAR M. HASTINGS
THOMAS R. AGG

Term expires January, 1946:

RICHARD E. DOUGHERTY
FRANKLIN THOMAS

DIRECTORS

Term expires January, 1945:

V. T. BOUGHTON
GEORGE W. BURPEE
SCOTT B. LILLY
A. M. RAWN
W. D. DICKINSON
J. T. L. McNEW

Term expires January, 1946:

R. E. BAKENHUS
DEAN G. EDWARDS
C. B. BREED
C. F. GOODRICH
N. W. DOUGHERTY
FRED C. SCOBEY

Term expires January, 1947:

S. C. HOLLISTER
GAIL A. HATHAWAY
RALEIGH W. GAMBLE
WILBUR M. WILSON
FRANK C. TOLLES
WILLIAM D. SHANNON
ROYCE J. TIPTON

PAST-PRESIDENTS

Members of the Board

E. B. BLACK
EZRA B. WHITMAN

TREASURER

CHARLES E. TROUT

SECRETARY

GEORGE T. SEABURY

ASSISTANT TREASURER

GEORGE W. BURPEE

ACTING ASSISTANT SECRETARY

JAMES E. JAGGER

PROCEEDINGS OF THE SOCIETY

SYDNEY WILMOT
Manager of Publications

HAROLD T. LARSEN
Editor of Proceedings

COMMITTEE ON PUBLICATIONS

SCOTT B. LILLY, *Chairman*

N. W. DOUGHERTY
S. C. HOLLISTER
FRED C. SCOBEY
WILBUR M. WILSON
MALCOLM PIRNIE, *ex officio*

Issue	Page Numbers
January	1– 126
February	127– 252
March	253– 426
April	427– 616
May	617– 790
June	791–1012
September	1013–1234
October	1235–1360
November	1361–1518
December	1519–1660

AMERICAN SOCIETY OF CIVIL ENGINEERS

COMING MEETING

BOARD OF DIRECTION MEETING

January 15–16, 1945:

A Quarterly Meeting will be held in New York, N. Y.

ANNUAL MEETING

NEW YORK, N. Y.

January 17–19, 1945

The Reading Room of the Society is open from 9:00 A.M. to 5:00 P.M. every day, except Saturdays when it is closed at 12:00 noon. It is closed all day on Sundays and holidays.

Members, particularly those from out of town, are cordially invited to use this room on their visits to New York, to have their mail addressed there, and to utilize it as a place for meeting others. There is an ample file of current periodicals, the latest technical books, and the room is well supplied with writing tables.